FrFT based OFDM system for Wireless Communication

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Abstract

The demand for higher data rates over wireless channels is growing day by day. This had provided by 4G systems by using multicarrier communication systems like OFDM. The BER of 4G systems can further improve by doing time, frequency analysis of signal while propagating in the channel. In this paper, Fractional Fourier transformation (FrFT) based OFDM system had presented with 512-PSK modulation technique. The BER response of proposed system had analysed under various wireless fading channels like AWGN, Rayleigh, Rician and Nakagami channels. The proposed system achieved improvement of 1.5 dB BER at 10⁻².⁹ SNR for $\alpha_{opt} = 0.9$ under the AWGN channel for 512-PSK modulation technique. For Rayleigh and Rician channel proposed system for 512-PSK modulation achieved 1.5 dB improvement in BER at 10⁻².⁹ SNR under both channels for $\alpha_{opt} = 0.4$ and $\alpha_{opt} = 0.9$ respectively. Under Nakagami channel, said system achieved 1.0 dB improvement in BER at 10⁻².⁸ SNR for $\alpha_{opt} = 0.4$ using the 512-PSK modulation technique.

Key Words: OFDM, FrFT, IFrFT, ICI, 512-PSK.

I. INTRODUCTION

In today’s world, the demand for higher data rate under the wireless domain had increased many folds, due to use of multimedia applications. Currently the 4G systems had deployed for satisfying the need of higher data rates with limited bandwidth. This had possible because of the use of the multicarrier communication. However, as there is scarcity of available spectrum, there is always a starvation for increasing speed with limited bandwidth. OFDM is the best solution, which has fulfilled this requirement [1]. OFDM had made possible the flow of higher data rates on wireless channels at a low symbol rate by using multipath transmission techniques. However, OFDM system was suffering from inter symbol interference and inter carrier interference. The definition of fractional Fourier transforms states it as the chirp basis expansion, which is defining the rotation in time frequency plane i.e. unified time frequency transformation. By changing the value of fractional constant $'\alpha'$ from 0.0 to 1.0, the signal characteristics can be transformed from time to frequency domain. So FrFT can also be said as a generalized form of Fourier transform (FT) [3]. FRFT is defined as:

$$F_{\alpha}\{x(t)\}(u) = X_{\alpha}(u) = \int_{-\infty}^{\infty} x(t) K_{\alpha}(t,u) dt$$

where

$$K_{\alpha}(t,u) = A_{\alpha} e^{j\pi(\sigma^2+u^2)\cot(\alpha/2)}e^{j\pi u^2}$$

and is called Transformation Kernel and $\alpha$ is the rotation angle of the transformed signal [4].

II. Proposed FrFT based OFDM system

In this new proposed OFDM model, FrFT and IFrFT had replaced with IFFT and FFT respectively at both transmitter and receiver and the improvement in performance of the receiver had achieved for modulation rate as high as up to 1024 in both M-PSK and M-QAM. Here by varying the fractional constant $\alpha$ the further improvement in BER could achieved for the given set of conditions [5-6]. The block diagram of the proposed OFDM system had shown in Fig. 1 below:
III. Response of fading channels for Multipath and Coherence Bandwidth

In mobile communication, receiver catches the signals from various paths. Due to reflection and diffraction, the various paths interfere with each other. Henceforth the output signal gets distorted. Further, a narrow band statistical process can calculate the random nature of the channel and complex valued Gaussian random process based on the limit theory. In addition, the baseband signal could calculate from the narrow band process as follows [7].

Let the transmitted signal had represented as

\[ X(t) = y(t) e^{2\pi ft} \]  \hspace{1cm} (1)

If the multipath follows a Gaussian scattering, then the channel response had modeled as

\[ R(\Gamma_n, t) = \sum_n \Delta n(\Gamma m, t) e^{-\pi Dn(\Gamma n(t))} \Delta \left[t - \Gamma_n(t)\right] \]  \hspace{1cm} (2)

where \( R(\Gamma n, t) \) is the response of the channel at time \( t \) due to an impulse applied at the time \( t - \Delta n(t) \)

\[ \Delta n(t) = \text{attenuation factor of the signal received on the } n^{th} \text{ path} \]

\[ \Gamma n(t) = \text{propagation delay for the } n^{th} \text{ path} \]

\[ D_n = \text{Doppler shift for the signal received on the } n^{th} \text{ path} \]

The Doppler shift had introduced due to the relative motion between the transmitter and the receiver and could written as

\[ D_n = \frac{v \cos \phi}{\lambda} \]

Where \( v \) the relative velocity between transmitter and receiver, \( \lambda \) is the wavelength of the carrier, and \( \phi \) is the phase angle between the transmitter and the receiver. The output of the transmit signal propagating through the channel can be written by multiplying eq. (1) and (2) given by

\[ U(t) = x(t) * R(\Gamma_n, t) \]  \hspace{1cm} (3)

\[ U(t) = y(t)e^{2\pi ft} \sum_n \Delta n(\Gamma m, t) e^{-\pi Dn(\Gamma n(t))} \Delta \left[t - \Gamma_n(t)\right] \]
Ankush Kansal, Kulbir Singh and Rajiv Saxena

\[
U(t) = \sum_n \left[ \epsilon_n(t) \right] e^{-j2\pi(f_n + D_n)\nu(t)} y(t - \Gamma_n(t)) e^{-j2\pi f_n t} \tag{4}
\]

where \( \Delta(t - \Gamma_n(t)) \) is a Gaussian random process. The density function of a Rayleigh faded channel is given by

\[
f(x) = \frac{x}{\Psi^2} e^{-\frac{x^2}{\Psi^2}} \tag{6}
\]

Channels without a direct line-of-sight (LOS) path had known as Rayleigh fading channel. Whereas the channel with a direct LOS path to the receiver was characterized by Rician density function called as Rician fading channel and written as [8]

\[
f(x) = \frac{x}{\Psi^2} B_x\left(\frac{x\bar{\tau}}{\Psi^2}\right) e^{-\frac{x^2 + \bar{\tau}^2}{2\Psi^2}} \tag{7}
\]

where \( B_x \) is the modified Bessel function of the 0th order and \( \bar{\tau} \) is mean and \( \Psi \) is variance of the direct LOS path [6] Autocorrelation function of \( R(\Gamma_n, t) \) is given as

\[
\Psi c(\Gamma, \nabla t) = H \left[ c(\Gamma, t)c^*(\Gamma, t + \nabla t) \right] \tag{8}
\]

It had measured by transmitting very narrow pulses and cross correlating the received signal with a conjugate delayed version of itself. The average power of the channel found by setting the \( \nabla = 0 \); that is, \( \Psi c(\Gamma, 0) = \Psi c(\Gamma) \). The quantity had known as the power delay profile. The range of values of \( \Gamma \) over which the \( \Psi c(\Gamma) \) is essentially non-zero is called the multipath delay spread of the channel, denoted by \( \Gamma_m \). The reciprocal of the multipath delay spread will measure the coherence bandwidth of the channel and is given by [6-8]

\[
BW = \frac{1}{\Gamma_m} \tag{9}
\]

The variation in channel bandwidth and Doppler shift could easily be equalized with the help of FrFT. Hence, the proposed model had been able to respond more efficiently for the OFDM as compared to traditional OFDM.

**IV. Performance analysis of Wireless Channels:**

In this paper performance analysis of four different wireless fading channels, namely AWGN, Rayleigh, Rician and Nakagami for the proposed OFDM system had presented. Further the model had been made to give the BER for different modulation techniques, namely 512-PSK [10-11], the various plots for different values of fractional constant \( \alpha \) in the range of 0.0 to 1.0 has been plotted and further analysis on the performance of the system based on these values of BER Vs SNR can be compared [12].

**4.1 Analysis for 512-PSK modulation technique for various wireless fading channels**

Here the analysis of BER vs. SNR of the different wireless fading channel on the proposed model for 512 PSK modulations for different values of \( \alpha \) in the range of 0.0 to 1.0 has done. Fig. 2 (a) to (f) shows the plots produced from the proposed model for BER vs. SNR for various channels.
Fig. 2 (a) Plot for 512-PSK in AWGN

Fig. 2 (b) Plot for 512-PSK in Rayleigh

Fig. 2 (c) Plot for 512-PSK in Rician

Fig. 2 (d) Plot for 512-PSK in Nakagami
Table 1 shows the improvement in BER achieved for the best value of $\alpha_{\text{opt}}$ for various wireless fading channels for 512-PSK calculated from the above shown plots. Father the improvement here has calculated by comparing the results of output with respect to the results of $\alpha = 1$ which is the case for FFT i.e. the traditional FFT based OFDM system.

Table 1: Improvement in SNR at $\alpha_{\text{opt}}$ w.r.t. $\alpha = 1$ for wireless fading channels for 512-PSK

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Rate of Modulation</th>
<th>$\alpha_{\text{opt}}$</th>
<th>Improvement in SNR (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>AWGN</td>
<td>0.9</td>
<td>1.5</td>
</tr>
<tr>
<td>2</td>
<td>Rayleigh</td>
<td>0.4</td>
<td>1.5</td>
</tr>
<tr>
<td>3</td>
<td>Rician</td>
<td>0.9</td>
<td>1.5</td>
</tr>
<tr>
<td>4</td>
<td>Nakagami</td>
<td>0.6</td>
<td>1.0</td>
</tr>
</tbody>
</table>

The best improvement in SNR comes out to be 1.5 dB for different wireless fading channels.

V. CONCLUSION

The results presented in this paper have shown that the proposed system achieved improvement of 1.5 dB BER at $10^{-2.9}$ SNR for $\alpha_{\text{opt}} = 0.9$ under the AWGN channel for 512-PSK modulation technique. For Rayleigh and Rician channel proposed system for 512-PSK modulation achieved 1.5 dB improvement in BER at $10^{-2.9}$ SNR under both channels for $\alpha_{\text{opt}} = 0.4$ and $\alpha_{\text{opt}} = 0.9$ respectively. Under Nakagami channel, said system achieved 1.0 dB improvement in BER at $10^{-2.8}$ SNR for $\alpha_{\text{opt}} = 0.4$ using the 512-PSK modulation technique.

REFERENCES

Authors Biography:

**Ankush Kansal** received his B. Tech. and M. Tech. Degree in Electronics and Communication Engineering from PTU, Jalandhar. He is currently pursuing Ph.D. from Thapar University, Patiala in the area of Wireless Communication. He is currently working as Assistant Professor in Thapar University, Patiala. He has published 30 research articles in referred international journals, international conference and national conference. He is a life time member of ISTE. His research interest includes Networking, Wireless Communication and Embedded Systems.

**Dr. Kulbir Singh** was born in Batala (Pb) India. He received his B.Tech degree in 1997 from PTU, Jalandhar. He obtained his ME and Ph.D. degree from Thapar Institute of Engineering and Technology, Patiala in 2000 and 2006 respectively. He worked as lecturer from 2000 to 2007 and as Assistant Professor from 2007 to 2011 in Electronics and Communication Engineering Department, Thapar University, Patiala. Presently he is working as Associate Professor in Thapar University, Patiala since July 2011. He has published about 60 research articles in refereed international journals, international conference and national conference. Best paper award of IETE was also conferred on him in the year 2008. He is life time member of IETE and ISTE. His research interests include Digital Signal Processing, Image Processing, Fractional Fourier Transform and Communication systems.

**Dr. Rajiv Saxena** joined IIT, Roorkee (erstwhile UOR, Roorkee), as a QIP Fellow, towards his Doctoral Degree Program. He holds Graduation (BE) and Post Graduation (ME) Degrees in Electronics and Telecommunication Engineering. The Ph. D. degree was conferred on him in Electronics and Computer Engineering. Professor Saxena served with Reliance, GRASIM, CIMMCO and Orimpex Industries and also had industrial experience of automation in process industry. He started his career as Lecturer in Electronics Engineering Department of Madhav Institute of Technology and Science, Gwalior. Where he became Professor in April 1997 and also worked as Head of the Department and Chairman, BOS. Professor Saxena moved, on lien from MITS, to Thapar Institute of Engineering and Technology (Now Thapar University), Patiala, as Professor in Electronics and Communication Engineering from June, 2000 to June 2002. He was the founder Head of the Department of Electronics and Communication Engineering at TIET, Patiala (Now Thapar University, Patiala). He was also Principal at Rustam Ji Institute of Technology, BSF Academy, Tekanpur, (again on lien from MITS Gwalior) for a period of two years from January, 2004 to January, 2006. Prof. Saxena also served as guest faculty at leading institutions, including ABV-IIITM, Gwalior, IITTM, Gwalior, and in M. Tech. Program of U. P. Technical University, Lucknow. Professor Saxena delivered various invited talks / keynote addresses at various platforms of repute. Prof. Saxena served as Coordinator of Project IMPACT for a period of two years (project was a joint venture of the World Bank and Ministry of Communication and IT, Government of India). Prof. Saxena executed two major research projects funded by AICTE besides the project under the FIST program of DST. Prof. Saxena has supervised THIRTEEN Ph.D. candidates in the area of digital signal processing, digital image processing and application of DSP tools in electronic systems. Prof. Saxena has published about 70 research articles in refereed journals of national and international repute. Best paper award of IETE had also conferred on him in the year 2008. Prof. Saxena is a Fellow of IETE and Sr. Member of CSI and ISTE.