

# A Review towards Evolutionary Multiobjective optimization Algorithms

Mr. Sunny Sharma

Assistant Professor, Department of  
Computer Science & Engineering, Guru  
Nanak Dev University, Amritsar, Punjab  
[sunnysharma05@yahoo.co.in](mailto:sunnysharma05@yahoo.co.in)

Dr. Rajinder Singh Virk

Associate Professor, Department of  
Computer Science & Engineering, Guru  
Nanak Dev University, Amritsar, Punjab  
[tovirk@yahoo.com](mailto:tovirk@yahoo.com)

## ABSTRACT

**Multi objective optimization is a promising field which is increasingly being encountered in many areas worldwide. Various metaheuristic techniques such as differential evolution (DE), genetic algorithm (GA), gravitational search algorithm (GSA), and particle swarm optimization (PSO) have been used to solve Multi objective problems. Various multiobjective evolutionary algorithms have been developed. Their principal reason for development is their ability to find multiple Pareto optimal solution in single run. Their Basic motive of evolutionary multiobjective optimization in contrast to single-objective optimization was optimality, decision making algorithm design (fitness, diversity, and elitism), constraints, and preference.**

**The goal of this paper is to trace the genealogy & review the state of the art of evolutionary multiobjective optimization algorithms.**

## 1. INTRODUCTION

The demand of multiobjective optimization in different fields like Bioinformatics, Engineering Design, Scheduling of Telecommunication network, Nuclear Power plants, Water Quality control growing massively. The term Optimization refers to finding the best solution or objective among the different set of constraints which may or may not conflict with each other. Since from 1970 various evolutionary multiobjective optimization algorithms has been developed like, VEGA, MOGA, NSGA, NPGA, WBGA, SPEA, SPEA2, SPEA2+, NSGA-II, R-NSGA-II, PAES, RWGA, PESA, RDGA, R-PESA-II, Multi-objective Evolutionary Algorithm

(MEA), Micro-GA, DMOEA, SEMO, FEMO, PISA Parallel and hybrid multi-objective GA they are the kind of refinement over each other under different set of problems & constraints. This paper will give overview on different evolutionary multiobjective optimization algorithms. In an attempt we avoid the unwieldy mathematical formulas and algorithms steps. We have just provided the facts & information regarding the Algorithm's. [2][3][5][7][8][18][31]

The multiobjective optimization theory remained relatively underdeveloped during the 1950s. It was until the 1960s that the foundations of multiobjective optimization were consolidated and taken seriously by pure mathematicians when Leonid Hurwicz generalized the results of Kuhn & Tucker to topological vector spaces. The application of multiobjective optimization to domains outside economics began with the work by Koopmans (1951) in production theory and with the work of Marglin (1967) in water resources planning. The first engineering application reported in the literature was a paper by Zadeh in the early 1960s. However, the use of multiobjective optimization became generalized until the 1970s. The potential of evolutionary algorithms in multiobjective optimization was hinted by Rosenberg in the 1960s, but the first actual implementation was produced in the mid-1980s (Schaffer, 1984). During ten years, the field remain practically inactive, but it started growing in the mid-1990s, in which several techniques and applications were developed.

Evolutionary algorithms seem particularly suitable to solve multiobjective optimization problems, because they deal simultaneously with a set of possible solutions (the so-called population) and make decision over it. This allows us to find several members of the Pareto



optimal set in a single run of the algorithm, instead of having to perform a series of separate runs as in the case of the traditional mathematical programming techniques. Where term Pareto optimal defines as a set of optimal trade-offs i.e. all objectives equally important and the term Decision making defines as to choose best compromise i.e. based on preference information. [2][3][5]

Additionally, evolutionary algorithms are less susceptible to the shape or continuity of the Pareto front e.g., they can easily deal with discontinuous or concave Pareto fronts, whereas these issues are a real concern for mathematical programming techniques. However, mathematical programming techniques have certain limitations when tackling MOPs. For example, many of them are susceptible to the shape of the Pareto front and may not work when the Pareto front is concave or disconnected.

In multiobjective problems if objectives conflict then many solution or set of solutions or tradeoffs will exist then we find compromised solution. If no conflict then single solution will exist. In nutshell it can be said that Multi-objective formulations are realistic models for many complex engineering optimization problems. In many real-life problems, objectives under consideration conflict with each other, and optimizing a particular solution with respect to a single objective can result in unacceptable results with respect to the other objectives.

A solution is said to be Pareto optimal if it is not dominated by any other solution in the solution space. A Pareto optimal solution cannot be improved with respect to any objective without decline at least one other objective. The set of all feasible non-dominated solutions in  $X$  is referred to as the Pareto optimal set, and for a given Pareto optimal set; the corresponding objective function values in the objective space are called the Pareto front. For many Problems, the number of Pareto optimal solutions is huge (perhaps infinite). [21][3][5]

The ultimate goal of a multi-objective optimization algorithm is to identify solutions in the Pareto optimal set. However, identifying the entire Pareto optimal set, for many multi-objective problems, is practically impossible due to its size. In addition, for many problems, especially for combinatorial optimization

problems, proof of solution optimality is computationally infeasible. Therefore, a practical approach to multi-objective optimization is to investigate a set of solutions that represent the Pareto optimal set. With these concerns in mind, a multi-objective maintaining a diverse population is an important consideration in multi-objective GA to obtain solutions uniformly distributed over the Pareto front. Without taking preventive measures, the population tends to form relatively few clusters in multi-objective GA. This phenomenon is called genetic drift, and several approaches have been devised to prevent genetic drift. There are many Multiobjective evolutionary algorithms present these days now let's move towards the journey of the various multi-objective evolutionary optimization algorithm. [21][3][5]

## 2. EVOLUTIONARY MULTIOBJECTIVE ALGORITHMS

**a) VEGA (Vector evaluated GA):-** it was proposed by Schaffer in 1985. It was first Multiobjective genetic algorithms which have straight forward implementation it deals with the Objective-wise selection, and each sub population is evaluated with respected to different objective, neither diversity mechanism and nor elitism was used also it is Non Pareto technique. It uses Fitness Assignment Strategies based on criterion based and it behaves as an aggregating approach the major drawback of VEGA was it tend to converge to the extreme of each objective.[2][3][18]

**b) MOGA (Multi-objective Genetic Algorithm):-** it was proposed by Fonseca & Fleming in 1993. It uses the Pareto-based selection similar to NSGA and NPGA. The Pareto-based Technique was Suggested by Goldberg (1989) to solve the problems with Schaffer's VEGA.

MOGA use the nondominated ranking and selection to move the population towards the pareto front. which requires a ranking procedure and a technique to maintain diversity in the population otherwise, the GA will tend to converge to a single solution, because of the stochastic noise involved in the process.



In MOGA methodology during its first pass it provides the highest rank to non dominated solution and next highest rank to non dominated solution among remaining ones i.e. The rank of a certain individual corresponds to the number of chromosomes in the current population by which it is dominated.

All nondominated individuals are assigned the highest possible fitness value. While dominated ones are penalized according to the population density of the corresponding region to which they belong i.e. fitness sharing is used to verify how crowded is the region surrounding each individual.

MOGA uses the Fitness Assignment Strategies of dominance-based so that it behaves as set-oriented & scaling-independent which allow evolutionary algorithms to maintain individuals among the nondominated frontier similar Fitness Assignment Strategies to be used in NSGA & SPEA. Its Dominance Based Ranking is dominance rank which tells by how many individuals is an individual dominated which is also similar to be used in NPGA. The Diversity Preservation is done by Density estimation technique through Kernel which is sum of  $f$  values where  $f$  is a function of the distance it is also used in NPGA. In NPGA Fitness sharing was choice but in MOGA Fitness sharing was not choice. [2][3][5][18]

**c) NPGA (Niche Pareto Genetic Algorithm):-**

It is proposed by Horn et al. in 1993/94). It uses a tournament selection scheme based on Pareto dominance. Two individuals are randomly chosen and they are compared against a subset from the entire population. When both competitors are either dominated or nondominated (i.e., when there is a tie), the result of the tournament is decided through fitness sharing in the objective domain a technique called equivalent class sharing was used in this case. It is easy to implement and efficient because it does not apply pareto ranking to the entire population. It seems to have a good overall performance.

Besides requiring a sharing factor, it requires another parameter which is tournament size. It use Niching technique and its Dominance Based Ranking is dominance rank which tells by how many individuals is an individual dominated which is similar to MOGA. The Diversity Preservation is done by Density estimation technique through Kernel which is sum of  $f$

values where  $f$  is a function of the distance. This Diversity Preservation technique is just similar to MOGA. [2][3][5][18][22]

**d) WBGA (Weight-based Genetic Algorithm):-**

it is proposed by Hajela and Lin in 1992. The classical approach to solve a multi-objective optimization problem is to assign a weight  $w_i$  to each normalized objective function  $z'_i(x)$  so that the problem is converted to a single objective problem with a scalar objective function as follows:

$$\text{Min } z = w_1 z'_1(x) + w_2 z'_2(x) + \dots + w_k z'_k(x) \quad (1)$$

Where  $z'_i(x)$  is the normalized objective function  $z_i(x)$  and  $\sum w_i = 1$ . This approach is called the priori approach since the user is expected to provide the weights. Solving a problem with the objective function (1) for a given weight vector  $w = \{w_1, w_2, \dots, w_k\}$  yields a single solution, and if multiple solutions are desired, the problem must be solved multiple times with different weight combinations. The main difficulty with this approach is selecting a weight vector for each run. To automate this process; Hajela and Lin [8] proposed the WBGA for multi-objective optimization (WBGA-MO) in the WBGA-MO, each solution  $x_i$  in the population uses a different weight vector  $w_i \{w_1, w_2, \dots, w_k\}$  in the calculation of the summed objective function (1). The weight vector  $w_i$  is embedded within the chromosome of solution  $x_i$ . Therefore, multiple solutions can be simultaneously searched in a single run. In addition, weight vectors can be adjusted to promote diversity of the population. [3][5][18]

**e) RWGA (Random Weighted Genetic Algorithm): -**

it is proposed in 1996. The tasks of RWGA are like WBGA. The RWGA is also likely to be unable to find Pareto optimum solution in non convex problems so difficult to find solution for uniformly distributed over a non convex trade off surface. The major advantage of RWGA is that the single objective can be achieved with min modification & it is computationally efficient. the major drawback of objective switching is that the population tends to converge to solutions which are superior in one objective, but poor at others. [3][18][22]

**f) NSGA (Nondominated Sorting Genetic Algorithm):-**

it is proposed by Srinivas and Deb in



1994. It is Pareto based technique just like MOGA & NPGA. It is based on several layers of classifications of the individuals. Nondominated individuals get a certain dummy fitness value and then are removed from the population. The process is repeated until the entire population has been classified. To maintain the diversity of the population, classified individuals are shared in decision variable space with their dummy fitness values. It is relatively easy to implement and seems to be very sensitive to the value of the sharing factor. It has been recently improved in NSGA II with elitism and a crowded comparison operator that keeps diversity without specifying any additional parameters. It did the Visual comparisons. It uses the Fitness Assignment Strategies of dominance-based so set-oriented, scaling-independent. This Fitness strategy is used in similar way in SPEA & MOGA. The Dominance Based Ranking is dominance depth which tells at which front is an individual located this ranking technique was similar to be used in NSGA-II.[2][3][18][22]

**g) SPEA (Strength Pareto Evolutionary Algorithm):-** The Eckart Zitzler and Thiele in 1999 introduce the algo SPEA which use elitism i.e it usually refers to the use of an external population which retain non dominated individuals in evolutionary process. The elitism set landmark in the field to use external population. The use of elitism is a theoretical requirement in order to guarantee convergence of an MOEA. Thus, what we need is a way of guaranteeing that the solutions that we will report to the user are nondominated with respect to every other solution that the algorithm has produced. The most intuitive way of doing this is by storing in an external memory or archive where all the nondominated solutions found. If a solution that wishes to enter the archive is dominated by its contents, then it is not allowed to enter. Conversely, if a solution dominates anyone stored in the file, the dominated solution must be deleted. For each individual in this external set, a strength value is computed. This strength is similar to the ranking value of MOGA, since it is proportional to the number of solutions to which a certain individual dominates. The fitness assignment process of SPEA considers both closeness to the true Pareto front and even distribution of solutions at the same time. Thus, instead of using

niches based on distance, Pareto dominance is used to ensure that the solutions are properly distributed along the Pareto front. Although this approach does not require a niche radius, its effectiveness relies on the size of the external nondominated set. In fact, since the external nondominated set participates in the selection process of SPEA, if its size grows too large, it might reduce the selection pressure, thus slowing down the search. Because of this, the authors decided to adopt a technique that prunes the contents of the external nondominated set so that its size remains below a certain threshold. It uses the Fitness Assignment Strategies of dominance-based so set-oriented, scaling-independent this Fitness strategy similar way to be used in NSGA & MOGA. Its Dominance Based Ranking is dominance count i.e. how many individuals does an individual dominate + dominance rank i.e. by how many individuals is an individual dominated. Which is similar to be used in SPEA2. It uses a ranking procedure to assign better fitness values to non-dominated solutions at underrepresented regions of the objective space. In SPEA, an external list  $E$  of a fixed size stores nondominated solutions that have been investigated thus far during the search. For each solution  $y \in E$ , a strength value is defined as

$$S(y,t) = np(y,t)/Np+1$$

where  $np(y,t)$  is the number solutions that  $y$  dominates in  $P$ . The rank  $r(y,t)$  of a solution  $y \in E$  is assigned as  $R_1(y,t) = s(y,t)$  and the rank of a solution is calculated as

$$R_1(y,t) = 1 + \sum_{y \in E, y > x} s(y,t).$$

Accumulated ranking density strategy also aims to penalize redundancy in the population due to over representation. This ranking method is given as.

$$R_2(y,t) = 1 + \sum_{y \in P, y > x} r(y,t).$$

Besides the use of an external file, elitism can also be introduced through the use of a  $(\mu + \lambda)$ -selection in which parents compete with their children. [2][3][5][18][22]

**h) SPEA2 (Improved SPEA):-** SPEA and SPEA2 are both very effective algorithms that use an external list to store non-dominated solution discovered so far in the search. They are also excellent examples for the use of external populations. Other examples of elitist approaches using external populations are PESA,



RDGA, RWGA, and DMOEA. SPEA2 has three main differences with respect to its predecessor SPEA they are. (1) it incorporates a fine-grained fitness assignment strategy which takes into account for each individual the number of individuals that dominate it and the number of individuals by which it is dominated (2) it uses a nearest neighbor density estimation technique for Diversity Preservation which guides the search more efficiently, (3) it has an enhanced archive truncation method that guarantees the preservation of boundary solutions. Its Dominance Based Ranking & Diversity Preservation is similar to the used in SPEA. The similar Diversity Preservation is also used in NSGA-II. A density measure is used to discriminate between solutions with the same rank, where the density of a solution is defined as the inverse of the distance to its  $k^{\text{th}}$  closest neighbor in objective function space. The density of a solution is similar to its niche count. However, selecting a value for parameter  $k$  is more straightforward then electing a value for  $\sigma_{\text{share}}$ . In cases where more than two objectives are present SPEA2 seems to signify some advantages over NSGA-II. [2][3][4][5][18][22]

**i) SPEA2+ (Improving the Performance of the Strength Pareto Evolutionary Algorithm 2):-** It is proposed by Mifa Kim, and others in 2004. Two new mechanisms were added to SPEA2 to improve its searching ability a more effective crossover mechanism and an archive mechanism to maintain diversity of the solutions in the objective and variable spaces. The new SPEA2 with these two mechanisms was named SPEA2+.objective and variable spaces. SPEA2+ adds the following operations to SPEA2.

1) Neighborhood crossover, which crosses over individuals close to each other in objective space. 2) Mating selection, which reflects all archived good individuals in the search? 3) Applying archive to allow holding of diverse solutions in the objective space and variable space.

By performing neighborhood crossover, population diversity can be obtained. By performing copy operation, solutions with better precision can be obtained. By using two archives, it is possible to obtain a wider variety of individuals in the variable space without affecting the search ability.

SPEA2+ which included the above operations, mostly showed better results than SPEA2 or NSGA-II. These

observations suggest that SPEA2+ is an effective algorithm.[3][4][5]

**j) PAES (Pareto-Archived Evolution Strategy):-** it is proposed by Knowles, Corne in 1999/2000. It uses Quantitative performance metrics. Its Diversity Preservation is done through Density estimation technique called histogram which tells number of solutions in the same box which is similar to PESA. This approach is very simple: it uses a (1+1) evolution strategy (i.e., a single parent that generates a single offspring). PAES with one parent and one child, the child is compared with respect to the parent. If the child dominates the parent, the child is accepted as the next parent and the iteration continues. On the other hand, if the parent dominates the child, the child is discarded and a new mutated solution (a new child) is found. However, if the child and the parent do not dominate each other, the choice between the child and the parent considers the second objective of keeping diversity among obtained solutions. To maintain diversity, an archive of non-dominated solutions is maintained. The child is compared with the archive to check if it dominates any member of the archive. If yes, the child is accepted as the new parent and the dominated solution is eliminated from the archive. If the child does not dominate any member of the archive, both parent and child are checked for their nearness with the solutions of the archive. If the child resides in a least crowded region in the parameter space among the members of the archive, it is accepted as a parent and a copy of added to the archive. Later, they suggested a multiparent PAES with similar principles as above. Authors have calculated the worst case complexity of PAES for  $N$  evaluations as  $O(amN)$ , where  $a$  is the archive length. Since the archive size is usually chosen proportional to the population size  $N$ , the overall complexity of the algorithm is  $O(mN^2)$ . Knowles & corne also present a memetic version of PAES called M-PAES; in this a set of local non-dominated solutions is used as a comparison set for solutions investigated during local search.

The (1+1) strategy may not be the best approach to find the Pareto set. more over most of the MOA use the concept of an archive that maintain a set of vectors non dominated among all decision vectors visited so far.



**k) PESA (Pareto Envelope-based Selection Algorithm):-** it is proposed by Knowles, Corne in 1999/2000. It Use Cell based density. In this approach the objective space is divided into K-dimensional cells. The number of solutions in each cell is defined as the density of the cell, and the density of a solution is equal to the density of the cell in which the solution is located. This density information is used to achieve diversity similarly to the fitness sharing approach. For example, in PESA, between two non-dominated solutions, the one with a lower density is preferable. It also uses Quantitative performance metrics. Its Diversity Preservation is done through Density estimation technique called histogram which tells number of solutions in the same box which is similar to PAES.[2][3][5][18][24]

**l) PESA-II (Region-based Selection in Evolutionary Multiobjective Optimization):-** PESA-II follows a more direct approach, namely region-based selection, where cells but not individual solutions are selected during the selection process. In this approach, a cell that is sparsely occupied has a higher chance to be selected than a crowded cell. Once a cell is selected, solutions within the cell are randomly chosen to participate to crossover and mutation.[2][3][5]

**m) NSGA-II (Fast Nondominated Sorting Genetic Algorithm):-** it is proposed by Deb et al. in 2000. It was improved version of NSGA & was developed to form non dominated fronts. it uses the few concepts of SPEA2 and NSGA. NSGA-II uses a fixed population size. The complete procedure of NSGA-II is can be given to demonstrate an implementation of elitism without using a secondary external population In the NSGA-II, for each solution one has to determine how many solutions dominate it and the set of solutions to which it dominates. It estimates the density of solutions surrounding a particular solution in the population by computing the average distance of two points on either side of this point along each of the objectives of the problem.(this value is called crowding distance). During selection, the NSGA-II uses a crowded comparison operator which takes into consideration both the nondomination rank of an individual in the population and its crowding distance (i.e., nondominated solutions are preferred over dominated solutions, but

between two solutions with the same nondomination rank, the one that resides in the less crowded region is preferred). The main advantage of the crowding approach described above is that a measure of population density around a solution is computed without requiring a user-defined parameter such as  $\sigma_{share}$  or the kth closest neighbor in NSGA-II, this crowding distance measure is used as a tie- breaker in a selection technique called the crowded tournament selection operator: Randomly select two solutions  $x$  and  $y$ ; if the solutions are in the same non-dominated front, the solution with a higher crowding distance is the winner. Otherwise, the solution with the lowest rank is selected Fonseca and Fleming used a slightly different rank assignment approach than the ranking based on non-dominated-fronts as follows  $r(x,t)=1+nq(x,t)$  where  $nq(x,t)$  is the number of solutions dominating solution  $x$  at generation  $t$ . This ranking method penalizes solutions located in the regions of the objective function space which are dominated (covered) by densely populated sections of the Pareto front. As the NSGA-II does not use an external memory as the other MOEAs. Instead, the elitist mechanism of the NSGA-II consists of combining the best parents with the best offspring obtained i.e., a  $(\mu + \lambda)$ -selection. Due to its clever mechanisms, the NSGA-II is much more efficient (computationally speaking) than its predecessor, and its performance is so good. Its Dominance Based Ranking is dominance depth which tells at which front is an individual located this ranking similar to be used in NSGA. Its Dominance Based Ranking & Diversity Preservation is similar to the used in SPEA2 but in later cases it uses crowding comparison operator. Note that when the combined parent and offspring population includes more  $N$  non-dominated solutions, NSGA-II becomes as a pure elitist GA where only non-dominated solutions participate in crossover and selection. The main advantage of maintaining non-dominated solutions in the population is straightforward implementation. In this strategy, the population size is an important GA parameter since no external archive is used to store discovered non-dominated solutions. In cases where more than two objectives are present the NSGA-II lacks behind then SPEA2. Most archiving strategies for NSGA-II & SPEA may loose pareto optimal solution



Another improvement done by **Kalyanmoy Deb, and others in their Report No. 200001** which named as **A Fast Elitist Non-Dominated Sorting Genetic Algorithm for Multi-Objective Optimization: NSGA-II**. The NSGA-II have been mainly criticized for their (i)  $O(mN^3)$  computational complexity (where  $m$  is the number of objectives and  $N$  is the population size), (ii) non-elitism approach, and (iii) the need for specifying a sharing parameter.

They suggest a non-dominated sorting based multi-objective evolutionary algorithm (we called it the Non-dominated Sorting GA-II or NSGA-II) which alleviates all the above three difficulties. Specifically, a fast non-dominated sorting approach with  $O(mN^2)$  computational complexity is presented. Second, a selection operator called crowded comparison operator ( $\geq_n$ ) is presented which guides the selection process at various stages of the algorithm towards a uniformly spread out pareto optimal front and it also creates a mating pool by combining the parent and child populations and selecting the best (with respect to fitness and spread)  $N$  solutions. Some Simulation results on five difficult test problems show that the proposed NSGA-II is able to find much better spread of solutions in all problems compared to PAES.

The diversity among non-dominated solutions is introduced by using the crowding comparison procedure which is used in the tournament selection and during the population reduction phase. Since solutions compete with their crowding distance (a measure of density of solutions in the neighborhood), no extra niching parameter (such as  $\sigma_{share}$  needed in the NSGA) is required here. Although the crowding distance is calculated in the objective function space, it can also be implemented in the parameter space, if so desired.

**Jinghua Zheng in 2012 proposed enhancing diversity for NSGA-II in evolutionary multi-objective optimization called SEEA(Sphere Excluding Evolutionary Algorithm):-** NSGA-II is that the diversity of resulting populations is not satisfactory due to the shortcoming of crowding distance. In this paper, they propose a diversity maintenance strategy for NSGA-II to enhance diversity during evolution process. They employ sphere to define a neighborhood for each individual. Moreover, a diversity maintenance strategy integrates into the critical selection scheme. It picks out extreme individuals and prohibits or postpones the

archive of adjacent individuals. From an extensive comparative study with original NSGA-II and two other MOEAs, the proposed method shows a good balance among convergence, uniformity and spread. They purpose the algorithm **SEEA (Sphere Excluding Evolutionary Algorithm)** which employs a diversity during evolution process and prevent the algorithm becoming trapped in a locally optimal Pareto front. They employ sphere to define a neighborhood for each individual. Thereafter they integrate a diversity maintenance strategy in to the critical selection scheme. It picks out extreme individuals and prohibits or postpones the archive of adjacent individuals.

**n) R-NSGA-II (Reference Point Based Multi-Objective Optimization Using Evolutionary Algorithms):-** it is proposed by Kalyanmoy Deb, J. Sundar, Udaya Bhaskara Rao N. and Shamik Chaudhuri in 2006. NSGA-II faces difficulty in solving problems with a large number of objectives. So R-NSGA-II try to remove that problem as they use the concept of reference point methodology in an EMO and attempt to find a set of preferred Pareto-optimal solutions near the regions of interest to a decision-maker. It shows another use of a hybrid-EMO methodology in allowing the decision-maker to solve multi-objective optimization problems better and with more confidence. In this they combine one such preference-based strategy with an EMO methodology and demonstrate how, instead of one solution, a preferred set of solutions near the reference points can be found parallelly. They propose two approaches for this task: (i) a modified EMO procedure based on the elitist non-dominated sorting GA or NSGA-II and (ii) a predator-prey approach based on original grid based procedure. On two-objective to 10-objective optimization test problems, the modified NSGA-II approach shows its efficiency in finding an adequate set of Pareto-optimal points. On two and three-objective problems, the predator-prey approach also demonstrates its usefulness. Such procedures will provide the decision-maker with a set of solutions near her/his preference so that a better and a more reliable decision can be made.[2][3][5][7][8][18][31]

**o) RDGA (Rank-Density Based Genetic Algorithm):-** Lu and Yen and Yen and Lu developed an efficient approach to identify a solution's cell in case of



dynamic cell dimensions. In this approach, the width of a cell along the  $k^{\text{th}}$  objective dimension is  $(z_k^{\text{max}} - z_k^{\text{min}})/n_k$  where  $n_k$  is the number cells dedicated to the  $k^{\text{th}}$  objective dimension and  $z_k^{\text{max}}$  and  $z_k^{\text{min}}$  are the maximum and minimum values of the objective function  $k$  so far in the search, respectively. Therefore, cell boundaries are updated when a new maximum or minimum objective function value is discovered. RDGA uses a cell-based density approach in an interesting way to convert a general bi-objective problem into a bi-objective optimization problem with the objectives to minimize the individual rank value and density of the population. It Use cell based density.

The main advantage of the cell-based density approach is that a global density map of the objective function space is obtained as a result of the density calculation. The search can be encouraged toward sparsely inhabited regions of the objective function space based on this map. RDGA uses a method based on this global density map to push solutions out of high density areas towards low density areas. Another advantage is its computational efficiency compared to the niching or neighborhood-based density techniques. Yen and Lu proposed several data structures and algorithms to efficiently store cell information and modify cell densities.[18]

**p) Parallel and hybrid multi-objective GA:-** All comparative studies on multi-objective GA agree that elitism and diversity preservation mechanisms improve performance. However, implementing elitism and diversity preservation strategies usually require substantial computational effort and computer memory. In addition, evaluation of objective functions may take considerable time in real-life problems. Therefore, researchers have been interested in reducing execution time and resource requirements of multi-objective GA using advanced data structures. One of the latest trends is parallel and distributed processing over multiple processors. Hybridization of GA with local search algorithms is frequently applied in single-objective GA. This approach is usually referred to as a memetic algorithm. Hybrid method ensures the convergence of an algorithm to pure Pareto front. Still There are some more evolutionary multiobjective optimization algorithms like Multi-objective Evolutionary Algorithm (MEA), Micro-GA, Dynamic Multi-objective Evolutionary Algorithm

(DMOEA), SEMO (Simple evolutionary multi objective optimizer), and FEMO (Fair evolutionary multi objective optimizer) [5][18][24]

**q) Reactive Search Optimization:-** it is proposed by Amir Mosavi, Atieh Vaezipour in 2012. They suggested that in solving real-life multiobjective optimization problems often most emphasis are spent on finding the complete pareto-optimal set and less on decision-making. However the complete task of multiobjective optimization is considered as a combined task of optimization and decision-making. They suggest an interactive procedure which will involve the decision-maker in the optimization process helping to choose a single solution at the end. Their proposed method works on the basis of Reactive Search Optimization (RSO) algorithms and available software architecture packages. [15]

**r) An Algorithmic Framework for Multiobjective Optimization:-** It is propose by T. Ganesan, and others in 2013 it describes that many challenges still arise especially when dealing with problems with multiple objectives (especially in cases more than two). In addition, problems with extensive computational overhead emerge when dealing with hybrid algorithms. T. Ganesan and others discuss these issues by proposing an alternative framework that utilizes algorithmic concepts related to the problem structure for generating efficient and effective algorithms. They propose a framework to generate new high-performance algorithms with minimal computational overhead for MO optimization.[16]

### 3. CONCLUSION

A plenty of methods have been used to find the optimal solution. Some of them deal with single objective wise selection & some with multi objective. To achieve the optimal multi objective solution the algorithms uses the pareto based technique based on dominance rank, dominance depth, pareto dominance etc. Some of them did the diversity preservation to find optimal solution. Diversity preservation using elitism plays significant role in many algorithms.





To find the optimal solution another parameter crowding comparison operator sets standard benchmark. This crowding comparison operator is used in Reference based NSGA-II algorithm which removes the flaws & reduce the complexity of NSGA. Recently SEEA enhanced the diversity of NSGA-II.

Some of the hybrid approaches are suggested but they did not show any advantages in overall computation time, but it is believed that the hybrid method may improve the ability of algorithm in finding the global optimal solutions.

All of the MOEA's tried their best for multiobjective optimization but not in overall decision making. The Reactive Search Optimization (RSO) describe complete task of multiobjective optimization. The RSO could be considered as a combined task of optimization and decision-making.

Another standard was set through algorithmic framework for multiobjective optimization. That was proposed to generate new high-performance algorithms with minimal computational overhead for multiobjective optimization.

In accordance to the demand of MOEA's in different fields it is observed that there is a crucial need of complete studies related to convergence of optimization problems & different ways of combining single and multi-objective methods to obtain the optimal result are required. The various algorithms already have been designed and they are performing their job very well but still there is a need to optimizing them so that we can find out some intensive & robust results.

#### 4. REFERENCES

- [1] Carlos A. Coello Coello, "CINVESTAV-IPN, MEXICO "Evolutionary Multi-objective optimization A Historical view of the Field", IN IEEE COMPUTATIONAL INTELLIGENCE MAGAZINE, FEBRUARY 2006,1556-603X/06/\$20.00 ©2006IEEE
- [2] Carlos A. Coello Coello," A Short Tutorial on Evolutionary Multiobjective Optimization", in Av. Instituto Polit'ecnico Nacional, No. 2508
- [3] Eckart Zitzler," Evolutionary Algorithms for Multiobjective Optimization" in Computer Engineering and Networks", in Lab Swiss Federal Institute of Technology (ETH) Zurich
- [4] Mifa Kim, Tomoyuki Hiroyasu, Mitsunori Miki, and Shinya Watanabe, "SPEA2+: Improving the Performance of the Strength Pareto Evolutionary Algorithm 2", in [Parallel Problem Solving from Nature - PPSN VIII ,Lecture Notes in Computer Science](#), Volume 3242, 2004, pp 742-751
- [5] R.T. Marler and J.S. Arora," Survey of multi-objective optimization methods for engineering", Struct Multidisc Optim 26, 369–395 (2004), DOI 10.1007/s00158-003-0368-6
- [6] Eckart Zitzler, Marco Laumanns, and Stefan Bleuler, "A Tutorial on Evolutionary Multiobjective Optimization", in [www.cs.cinvestav.mx/~emooworkgroup/zitzler04.pdf](http://www.cs.cinvestav.mx/~emooworkgroup/zitzler04.pdf)
- [7] Kalyanmoy Deb, Samir Agrawal, Amrit Pratap, and T Meyarivan,"A Fast Elitist Non-Dominated Sorting Genetic Algorithm for Multi-Objective Optimization: NSGA-II)", in KanGAL Report No. 200001
- [8] C.A. Coello Coello, "A Comprehensive survey of evolutionary-based multiobjective optimization techniques," Knowledge and Information Systems. An International Journal, vol. 1, no. 3, pp. 269–308, Aug. 1999.
- [9] C.A. Coello Coello and G.B. Lamont, editors. Applications of Multi-Objective Evolutionary Algorithms. World Scientific, Singapore, 2004. ISBN 981-256-106-4.
- [10] K. Deb, S. Agrawal, A. Pratab, and T. Meyarivan, "A fast elitist non-dominated sorting genetic algorithm for multi-objective optimization: NSGA-II," In Marc Schoenauer et al., editor, Proceedings of the Parallel Problem Solving from Nature VI Conference, pp.849–858, Paris, France, 2000. Springer. Lecture Notes in Computer Science No. 1917.
- [11] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, "A fast and elitist multiobjective genetic algorithm: NSGA-II," IEEE Transactions on Evolutionary Computation, vol. 6, no. 2, pp. 182–197, Apr. 2002.
- [12] E. Zitzler, M. Laumanns, and L. Thiele, "SPEA2: Improving the strength pareto evolutionary algorithm," In K. Giannakoglou et al., editor, EUROGEN 2001. Evolutionary Methods for Design, Optimization and Control with Applications to Industrial Problems, pp. 95–100, Athens, Greece, 2002.
- [13] E. Zitzler and L. Thiele, "Multiobjective evolutionary algorithms: A comparative case study and the strength pareto approach," IEEE Transactions on



- Evolutionary Computation, vol. 3, no. 4, pp. 257–271, Nov. 1999.
- [14] Priti Punia, Maninder Kaur, "Various Genetic Approaches for Solving Single and Multi-Objective Optimization Problems: A Review" in International Journal of Advanced Research in Computer Science and Software Engineering, Volume 3, Issue 7, July 2013, ISSN: 2277 128X
- [15] Amir Mosavi, Atieh Vaezipour, Reactive Search Optimization; Application to Multiobjective Optimization Problems Applied Mathematics, 2012, 3, 1572-1582 <http://dx.doi.org/10.4236/am.2012.330217> Published Online October 2012 (<http://www.SciRP.org/journal/am>)
- [16] T. Ganesan, I. Elamvazuthi, Ku Zilati Ku Shaari, and P. Vasant, "An Algorithmic Framework for Multiobjective Optimization" IN Hindawi Publishing Corporation The Scientific World Journal Volume 2013, Article ID 859701, 11 pages <http://dx.doi.org/10.1155/2013/859701>
- [17] Kalyanmoy Deb "Multi-Objective Optimization Using Evolutionary Algorithms: An Introduction" in KanGAL Report Number 2011003, February 10, 2011
- [18] Abdullah Konak, David W. Coit, Alice E. Smith, "Multi-objective optimization using genetic algorithms: A tutorial" in [www.elsevier.com/locate/ress](http://www.elsevier.com/locate/ress), Reliability Engineering and System Safety 91 (2006) 992–1007
- [19] Zitzler E, Laumanns M, Thiele L. SPEA2: improving the strength Pareto evolutionary algorithm. Swiss Federal Institute Technology: Zurich, Switzerland; 2001.
- [20] Zitzler E, Deb K, Thiele L. Comparison of multiobjective evolutionary algorithms: empirical results. Evol Comput 2000;8(2):173–95. [11] Zitzler E, Thiele L. Multiobjective evolutionary algorithms: a comparative case study and the strength Pareto approach. IEEE Trans Evol Comput 1999;3(4):257–71.
- [21] Coello CAC. A comprehensive survey of evolutionary-based multiobjective optimization techniques. Knowl Inform Syst 1999;1(3):269–308.
- [22] Coello CAC. An updated survey of evolutionary multiobjective optimization techniques: state of the art and future trends. In: Proceedings of the 1999 congress on evolutionary computation-CEC99, 6–9 July 1999. Washington, DC, USA: IEEE.
- [23] Coello CAC. An updated survey of GA-based multiobjective optimization techniques. ACM Comput Surv 2000;32(2):109–43.
- [24] Fonseca CM, Fleming PJ. Genetic algorithms for multiobjective optimization: formulation, discussion and generalization. In: Proceedings of the ICGA-93: fifth international conference on genetic algorithms, 17–22 July 1993. Urbana-Champaign, IL, USA: Morgan Kaufmann; 1993.
- [25] Fonseca CM, Fleming PJ. Multiobjective optimization and multiple constraint handling with evolutionary algorithms. I. A unified formulation. IEEE Trans Syst Man Cybern A 1998;28(1): 26–37.
- [26] Jensen MT. Reducing the run-time complexity of multiobjective EAs: The NSGA-II and other algorithms. IEEE Trans Evol Comput 2003;7(5):503–15.
- [27] Xiujuan L, Zhongke S. Overview of multi-objective optimization methods. J Syst Eng Electron 2004;15(2):142–6.
- [28] Coello CAC. 2005, <http://www.lania.mx/ccello/EMOO/EMOObib.Html>
- [29] Kalyanmoy Deb, J. Sundar, Udaya Bhaskara Rao N. and Shamik Chaudhuri "Reference Point Based Multi-Objective Optimization Using Evolutionary Algorithms" in International Journal of Computational Intelligence Research. ISSN 0973-1873 Vol.2, No.3 (2006), pp. 273–286
- [31] Jinghua Zheng, Ruimin Shen, Juan Zou, "Enhancing diversity for NSGA-II in evolutionary multi-objective optimization" in 2012 8<sup>th</sup> International Conference on Natural Computation (ICNC 2012)

