ORTHOGONAL RADIAL MOMENTS, TRANSFORMS AND THEIR APPLICATIONS

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Abstract: Various radial moments i.e. Zernike moments, Pseudo-Zernike moments, Orthogonal Fourier Mellin moments, Chebyshev Harmonic moments, Radial Harmonic Fourier moments and Polar Harmonic Transforms such as polar complex exponential transforms, polar cosine transforms and polar sine transforms satisfy the condition of orthogonality. Therefore these moments and transforms possess minimum information redundancy and having good characteristic of image representation. In this paper we discuss about the various orthogonal radial moments, transforms and their applications in image processing.

Index terms: *Radial moments, Zernike moments, Pseudo-Zernike moments, Orthogonal Fourier Mellin moments, Radial Harmonic Fourier moments, Chebyshev's Harmonic Fourier moments, Polar Harmonic Transforms, orthogonal moments.*

1. INTRODUCTION

Moments are the scalar quantities used to characterize a function and to capture its significant feature. In the mathematical form moments are projection of a function onto a polynomial basis function. Moments computed from a digital image gives information about geometrical features of the image. Moments are of two types orthogonal and non orthogonal. Orthogonal moments specify independent features of the image, therefore, their information redundancy is minimum. Orthogonal moments have different applications in image analysis and pattern recognition due to their ability to represent image features. These moments include Zernike moments (ZMs), Pseudo-Zernike moments (PZMs), Orthogonal Fourier Mellin moments (OFMMs), Radial Harmonic Fourier moments (RHFMs) and Chebyshev's Harmonic Fourier moments (CHFMs). Teague[1] presented a set of orthogonal Zernike moments which are less sensitive to noise and has rotation invariance property in continuous domain. ZMs are widely used in pattern recognition applications like image reconstruction, image segmentation, edge detection, face recognition etc. Bhatia and Wolf [2] introduced another class of orthogonal radial moments called PZMs. ZMs and PZMs have similar characteristics of their minimum information redundancy and immunity to noise. Sheng and Shen [3] introduced another set of rotational invariant moments called OFMMs. OFMMs have better performance than ZMs and PZMs terms of noise sensitivity for small images. OFMMs give more number of moments as compared to ZMs and PZMs for the same maximum moment order. Because the repetition parameter *q* in Zernike and pseudo Zernike polynomials is not independent. Orthogonal radial moments are computationally intensive and numerically instable at higher order of moments. In order to minimize these problems Radial Harmonic Fourier moments (RHFMs) [4] are introduced, which are also orthogonal. These moments face the problem of numerical stability, when the higher order moments are concerned i.e. at higher order of moments or transform the quality of reconstructed images deteriorates significantly which is due to numerical instability in moments and transforms. The problem of these moments is related to the fact that many factorial terms are involved in the process of calculating the moment kernels. In order to overcome this problem Yep et al. [5] introduced various Polar

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Harmonic Transforms (PHTs) such as Polar Complex Exponential Transforms (PCETs), Polar Cosine Transforms (PCTs), and Polar Sine Transforms (PSTs). Computation of Polar Harmonic Transform (PHT) kernels is simple as compare to ZMs and PZMs, hence can be performed at a higher speed and there is no numerical instability issue.

These moments and transforms suffer from various errors and numerical instability for high order of moments. The common errors effect on the image reconstruction. The radial moments are computed by mapping a rectangular or square image geometrically inside a unit circle. Geometric error and numerical integration error occurs by using this mapping. In this mapping only those pixels are involved in moment computation whose centers lie inside the unit disc, while discarding other pixels. To eliminate the geometric error, Chong et.al introduced an alternative approach in which the complete image is contained inside the unit disc. All pixels are included in the computation of radial moments. Numerical integration error occurs when the double integration is included in the moment computation is approximated by summation. Numerical instability is another major error in calculation of radial moment which arises due to high orders of moments. Numerical instability is arises due to the high order factorial terms present in radial polynomials. At high orders of moments images reconstructed using PZMs, OFMMs, CHFMs, RHFMs produce error at the centre of the reconstructed image. But ZMs have best image reconstruction capability then PZMs, OFMMs, CHFMs, RHFMs.

2. MATHEMATICAL FORMULATION OF MOMENTS AND TRANSFORMS

2.1 Zernike Moments

Zernike introduces a set of complex polynomials which form a complete orthogonal set over the interior of the unit circle, i.e. $x^2 + y^2 \le 1$.The two dimensional Zernike moment of order p with repetition q over a unit disc is given by [1]:

$$Z_{pq} = \frac{p+1}{\pi} \iint_{x^2+y^2 \le 1} f(x,y) V_{pq}^{*}(x,y)\, dxdy.$$

(1)

where

p positive integer or zero

q positive and negative integers subject to constraints $p - |q| =$ even, $|q| \le p$

The functions $V_{pq}^{*}(x,y)$ are the complex conjugate of Zernike polynomial $V_{pq}(x,y)$ which is orthogonal and complete. It is defined as

$$V_{pq}(x,y) = R_{pq}^{Z}\ (x,y)\, e^{jq\theta}$$

(2)

$\theta = \tan^{-1}(y/x)$, $\theta \in [0, 2\pi]$, $j = \sqrt{-1}$

Radial polynomial of ZMs is given by

$$R_{pq}^{Z}\ (x,y) = \sum_{s=0}^{(p-|q|)/2} \frac{(-1)^s (p-s)!\ (x^2+y^2)^{\frac{p-2s}{2}}}{s!\left(\frac{p+|q|}{2}-s\right)!\left(\frac{p-|q|}{2}-s\right)!}$$

(3)

Zernike moments with negative values of repetition *q* are obtained directly by making use of the complex conjugate of Zernike moments for positive values.

2.2 PSEUDO ZERNIKE MOMENTS (PZMS)

Pseudo Zernike moments are also defined inside the unit circle. The two dimensional PZMs order p and repetition q of an image function $f(x,y)$ over a unit disk are defined by [6]:

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$$A_{pq} = \frac{p+1}{\pi} \iint_{x^2+y^2 \le 1} f(x,y) V^*_{pq}(x,y)\, dxdy.$$
(4)

The function $V^*_{pq}(x,y)$ is the complex conjugate of Pseudo Zernike polynomial $V_{pq}(x,y)$ which is orthogonal and complete. It is defined as

$$V_{pq}(x,y) = R^{P}_{pq}\ (x,y)\, e^{jq\theta},$$
(5)

where p is a non-negative integer and q is an integer, $|q| \le p$, $\theta = \tan^{-1}(y/x)$,

$\theta \in [0, 2\pi]$, $j = \sqrt{-1}$ and

$$R_{pq}(x,y) = \sum_{s=0}^{p-|q|} \frac{(-1)^s (2p+1-s)!\,(x^2+y^2)^{\frac{p-s}{2}}}{s!\,(p+|q|+1-s)!\,(p-|q|-s)!}\ .$$
(6)

2.3 Orthogonal Fourier- Mellin Moments (OFMMs)

Orthogonal Fourier Mellin moments have better performance than ZM's and PZM's in terms of image description and noise sensitivity in case of small images. For an image function f(x,y) the two dimensional OFMMs of order *p* with repetition *q* is given by [3]:

$$O_{pq} = \frac{p+1}{\pi} \iint_{x^2+y^2 \le 1} f(x,y) V^*_{pq}(x,y) dxdy$$
(7)

where *p* is a non-negative integer, *q* is an integer (negative or positive) and $V^*_{pq}(x,y)$ is the complex conjugate of the moment basis function $V_{pq}(x,y)$ and the basis function $V_{pq}(x,y)$ is orthogonal to each other. It is defined as

$$V_{pq}\ (x,y) = R^{O}_{pq}\ (r)\, e^{jq\theta},$$
(8)

where $r = \sqrt{x^2+y^2}$, $\theta = \tan^{-1}(y/x)$, $j = \sqrt{-1}$ and the radial polynomial $R^{O}_{pq}(r)$ is given by

$$R^{O}_{pq}(r) = \sum_{s=0}^{p} \frac{(-1)^{p+s}\,(p+s+1)!\,r^s}{s!\,(s+1)!\,(p-s)!}$$
(9)

Orthogonal Fourier Mellin Polynomials (OFMPs) are independent of *q*, there is no restriction on *q* while computing OFMMs, unlike ZMs and PZMs. In fact, the PZPs become OFMPs if *q*=0 is substituted in PZPs.

2.4 Radial Harmonic Fourier Moments (RHFMs)

The orthogonal radial moments are computational intensive and numerically instable at higher orders of moments. Therefore to overcome or minimize these problems Radial Harmonic Fourier Moments (RHFMs) are use which are also orthogonal.

The RHFMs of order p and repetition q with $p \ge 0$ *and* $|q| \ge 0$ over a unit disk are defined as [4]:

$$M_{pq} = \frac{1}{2\pi} \int_0^{2\pi} \int_0^1 f(r,\theta) V^*_{pq}(r,\theta)\, r dr d\theta$$
(10)

where $V^*_{pq}(r,\theta)$ is the complex conjugate of $V_{pq}(r,\theta)$ of order p and repetition q , and

$$V_{pq}(r,\theta) = R_p\ (r)\, e^{jq\theta}$$
(11)

The radial kernel functions are defined by

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$$R_p(r) = \begin{cases} \dfrac{1}{\sqrt{r}}, & p = 0 \\ \sqrt{\dfrac{2}{r}}\ \cos(\pi p r), & p\ even \\ \sqrt{\dfrac{2}{r}}\ \sin(\pi(p+1)r), & p\ odd \end{cases}$$

(12)

The orthogonal property for radial kernel is given as:

$$\int_0^1 R_p(r)\, R_k(r)\, rdr = \delta_{pk}$$

(13)

The orthogonality of basis function is given as:

$$\int_0^{2\pi}\int_0^1 V_{pq}(r,\theta) V^*_{p'q'}(r,\theta)\, rdrd\theta = 2\pi\, \delta_{pp'} \delta_{qq'}$$

(14)

2.5 Chebyshev-Fourier Moments (CHFMs)

Chebyshev-Fourier moments (CHFMs) exhibiting almost similar performance as OFMMs for describing the images [7]. CHFMs of order p and repetition q with $p \geq 0$ *and* $|q| \geq 0$ are defined in polar form as:

$$\Psi_{pq} = \frac{1}{2\pi}\int_0^{2\pi}\int_0^1 f(r,\theta) V^*_{pq}(r,\theta)\, rdrd\theta$$

(15)

where $V^*_{pq}(r,\theta)$ is the complex conjugate of the basis function $V_{pq}(r,\theta)$ defined by:

$$V_{pq}(r,\theta) = R_p\ (r)\, e^{jq\theta}$$

(16)

where the radial kernel function is given as:

$$R_p(r) = \sqrt{\frac{8}{\pi}}\left(\frac{1-r}{r}\right)^{1/4} \sum_{k=0}^{\lfloor p/2 \rfloor} (-1)^k \frac{(p-k)!}{k!\,(p-2k)!} \times \left(2(2r-1)\right)^{p-2k}$$

(17)

2.6 Polar Harmonic Transforms

Polar harmonic transforms (PHTs) are orthogonal rotation invariant transforms that provide many numerically stable features as the kernel computation of PHTs is very simple and is numerically stable. Three different transforms namely Polar Complex Exponential Transform (PCET), Polar Cosine Transform (PCT), and Polar Sine Transform (PST) are grouped under PHTs. These transforms have also low time complexity and numerical stability. For an image function $f(r,\theta)$ the two dimensional *PHTs* of order p with repetition q are defined as

$$M_{pq} = \mu \int_0^{2\pi}\int_0^1 f(r,\theta) H^*_{pq}(r,\theta)\, rdrd\ \theta$$

(18)

where

$$\begin{cases} |p| \geq 0, |q| \geq 0\ and\ \mu = \dfrac{1}{\pi}, & PCET \\ p \geq 0, |q| \geq 0\ and\ \mu = \dfrac{1}{\pi}\ \ if\ \ p = 0\ ,\ \mu = \dfrac{2}{\pi}\ \ if\ \ p \neq 0, & P \\ p \geq 1, |q| \geq 0\ and\ \mu = \dfrac{2}{\pi}, & PST \end{cases}$$

and $H^*_{pq}(r,\theta)$ is the complex conjugate of the basis function $H_{pq}(r,\theta)$. For *PCET*, $H_{pq}(r,\theta)$ is defined as

$$H_{pq}(r,\theta) = R_p(r)\, e^{jq\theta}$$

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where $R_p(r)$ is radial kernel, $j=\sqrt{-1}$, $\theta=\tan^{-1}(y/x)$. Similar to *PCET*, the basis functions of *PCT* and *PST* are defined respectively as

$$H_{pq}(r,\theta)=R_p^C(r)\,e^{jq\theta} \quad (20)$$

$$H_{pq}(r,\theta)=R_p^S(r)\,e^{jq\theta} \quad (21)$$

and their radial kernels are

$$R_p(r)=e^{i2\pi pr^2},\ R_p^C(r)=\cos(\pi p\, r^2),\ R_p^S(r)=\sin(\pi p\, r^2)$$

3. APPLICATIONS OF ORTHOGONAL RADIAL MOMENTS AND TRANSFORMS

There are various applications of orthogonal radial moments and transforms viz. to extract the features and the contents (color, texture, shape etc.) of medical imaging (tumor detection, cancer detection), fingerprint recognition, face recognition, data compression etc.

3.1 Medical Image Analysis

Medical image analysis is the technique and process of solving or analyzing medical problems and creating a picture of inside (interior) of a body for clinical purpose and medical action. Medical Images are based on different imaging modes and digital image analysis techniques. Medical imaging finds to make known the structures which are not visible by the skin and bones and also to identify and cure the disease. The most important field in medical image processing is medical image visualization and advances in analysis of various methods for example, an essential part of the early detection, diagnosis, and treatment of different diseases like cancer, brain tumor etc. The aim of medical image analysis is to find out the relevant information or knowledge from medical images. The different methods can be grouped into several broad categories: image segmentation, image registration, detection, noise reduction and others. The main challenge of medical image analysis is to process and analyze effectively the medical images to extract, quantify, and interpret this information to understand and insight into the structure and function of the organs being imaged and put it to for practical use. Medical images provide important information about the disease which is based on the macroscopic, microscopic, biochemical and molecular examination of organs and tissues. The growth of Medical images in database is very high in the past few years when the medical digital image equipments such as CT (**Computed tomography)**, MRI **(Magnetic resonance imaging)** are used in the clinic works. Iscan [8] et. al. proposed 2D continuous wavelet transform(CWT) for feature extraction, determination of asymmetry by using moments and detection of the tumor. Transforms has great potential in medical image compression and edge detection. A.S.Tolba [9] discovers the best design parameters for a data compression scheme applied to medical images of different imaging modalities. This technique reduces the transmission cost while preserving the diagnostic integrity. Pseudo-Zernike moments [10] of an image are used for shape descriptor, which have better features representation capabilities and are more robust to noise than other moment representations.

3.2 Fingerprint Recognition

Fingerprints recognition refers to the automated method of verifying a match

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between two fingerprints of the same person. A fingerprint recognition system can be used for both verification and identification. The range of applications also includes Banking Security (ATM Security), prison visitors, identification of missing children etc. The steps for fingerprint recognition include image acquisition, preprocessing, feature extraction and matching. Zernike moments plays important role in fingerprint recognition. ZMs are used for feature extractor due to its robustness to noise and orthogonal properties. ZMs are less sensitive to noise and with superior image representation capability. ZMs are very useful for understanding the image. ZMs are used in pattern recognition applications and image reconstruction. PZMs are used to create invariant feature vectors for the fingerprint biometric. It has rotation invariant properties. PZMs are used to extract global features of the image. PZMs extract image features independently with less information redundancy. These reasons make PZMs more desirable for image recognition. PZMs have important properties such as robust to noisy images and good image reconstruction capability. Orthogonal Fourier Mellin moments have better performance than ZM’s and PZM’s in terms of image description and noise sensitivity in case of small images. Polar harmonic transforms (PHTs) have been used in image processing applications. PHTs have various applications such as fingerprint classification, pattern recognition, character recognition, image reconstruction, etc. PHTs are computationally fast. These transforms have properties such as rotation invariant features and numerical stability. PHTs are used where maximal discriminant information is needed. The computation of PHTs kernel is simpler as compared to Zernike moments (ZMs) and pseudo Zernike moments (PZMs).

3.3 Face Recognition

Face Recognition is a pattern recognition task performed specifically on faces and it can be described as classifying a face either "known" or "unknown", after comparing it with stored known individuals. Face recognition is an important and active area of research in digital image processing due to the complexity of face recognition problem and its wide applications in many fields. These applications range from static matching of controlled format photographs such as passports, credit cards, photo IDs driver's licenses etc. The range of applications also includes building or office security, criminal identification and authentication in secure systems like computers or bank teller machines. There are three stages in face recognition problems, namely face segmentation from an image, feature extraction and classification. The stage of feature extraction is very important as it ultimately determines the performance of a recognition system. Although there are standard methods for classification and many researchers use them to test the efficiency of image (face) features derived by them. Thus the stage of feature extraction becomes very important. The existing feature extraction or face representation techniques fall into two categories –local based and global based. The first one is based on extraction geometrical and structural facial features such as shapes of eyes, nose, mouth and distance between these. These methods are not affected by irrelevant information in the image, but are sensitive to unpredictability of face appearance and environmental conditions. In the second approach which is statistical based methods, features from whole image are extracted. Since the features so derived represent the whole image, they are less sensitive to image noise and geometrical distortions. However, their

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performance is impaired by irrelevant elements like background effects. Since faces may undergo geometric transformations such as rotation, scaling, translation, the orthogonal radial moments provide the desired characteristics of being invariant to such changes. Among the widely used radial orthogonal moments include the Zernike moments, pseudo Zernike moments and orthogonal Fourier Mellin moments. In the recent past wavelet moments and wavelet transform have also been used for feature extraction and hybrid approaches combining features from different approaches are also in use.

4. CONCLUSION

There are number of orthogonal radial moments and harmonic transforms that are rotation invariant. These moments and transforms are made scale invariant when they are computed in a unit disc. Due to their unique characteristic of being rotation and scale invariant, these moments and transforms have been used various image processing applications. With the further development of orthogonal radial moments and transforms, they will be widely applied to the domain of image processing.

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