Opposition aided Cat Swarm Optimization Algorithm for Digital IIR Low Pass Filter Design

Kamalpreet Kaur Dhaliwal[1a], Jaspreet Singh Dhillon[1b]

[1]Department of Electrical and Instrumentation Engineering, SantLongowal Institute of Engineering and Technology, Longowal, Punjab, INDIA

E-mail: [a]kamal_dhaliwal12@yahoo.co.in(Corresponding author), [b]jsdhillonp@yahoo.com

Abstract. This paper undertakes the designing of optimal and stable digital infinite impulse response (IIR) low pass (LP) filter by employing the cat swarm optimization (CSO) technique with oppositional learning. CSO is a population based global optimization technique which possesses global as well as local search capabilities. The conventional optimization techniques used to design the digital IIR filter generally got caught in the local minima as the error surface of the digital IIR filter is non linear and multimodal because of the presence of the denominator terms. Although, CSO possesses better parameter estimation and has a much higher convergence speed than genetic algorithm and particle swarm optimization algorithm, it requires a higher computation time because the local and global searches are carried out independently in each iteration. So, in order to reduce the computational time this paper attempts to incorporate the concept of opposition based learning (OBL) strategy. The main idea behind OBL is the simultaneous consideration of an estimate and its corresponding opposite estimate in order to achieve a better approximation for the current candidate solution. The proposed opposition based cat swarm optimization method starts with some initial random solutions that are improved by moving towards optimal solution. The computation time is improved by starting with a better solution by simultaneously checking the opposite solution in the search space. Here, the multicriterion optimization is utilized as the design criterion that undertakes the minimization of magnitude approximation error and minimization of ripple magnitudes while satisfying the stability constraints that are imposed during the design process. The developed algorithm strives to find the optimal filter coefficients which are approximately close to the desired filter response. The computational results reveal that the proposed algorithm is capable of designing the stable and optimal digital IIR LP filter structure that is superior to the designs presented by other algorithms and can also be efficiently applied for the design of higher order LP filter.

Keywords: Digital IIR filter, cat swarm optimization algorithm, opposition based learning, low pass filter design, multiparameter optimization.

1. Introduction

Filtering is a process of removing the unwanted portion such as noise from the signal and to extract the useful information out of it [1]. Digital filters are widely employed in all signal processing applications like noise reduction, telecommunication, radars, channel equalization, speech synthesis, biomedical signal processing etc. Based upon the impulse response the digital filters can be classified as Finite impulse response (FIR) filters and infinite impulse response (IIR) filters. The digital FIR filter has impulse response of finite duration and its output is calculated from the present and past input values only. Hence, these filters are said to be non-recursive. On the other hand, the impulse response of IIR filter is continues or infinite and its output depend upon the present and past input values as well as on the previous output values. Hence, they are termed as recursive filters [2]. The digital IIR filters involve feedback because of the dependence of the present output on the previous output values. Digital IIR filters have become the target of growing interest as they provide much better performance, improved selectivity and less computational cost than the FIR filters for similar magnitude specifications. Also, they have a much sharper roll-offs in their frequency responses than the FIR filters of equal complexity.

The digital IIR filter designing mainly follows two approaches, namely: (i) transformation approach and (ii) optimization approach. The transformation approach involves the transformation of an analog filter to a digital filter for a given set of prescribed specifications [3]. But the performance of digital IIR filters designed by using the transformation approach is not good as they require too much pre-knowledge and return a single solution in most of the cases. Therefore, various optimization methods have been proposed to obtain optimal filter performances, where the magnitude approximation error, mean-square-error, and ripple magnitudes of both pass band and stop band are generally used as criteria to measure the performance of the designed digital IIR filters. In order to overcome the shortcomings of conventional methods and to achieve a global optimal solution, in the past years many nature inspired optimization algorithms have been implemented for the digital IIR filter design problem. Under the optimization approach various methods like the direct search and the gradient search methods have been proposed. IIR filters are generally multimodal with respect to the filter coefficients and the conventional gradient-based algorithm easily stuck at local minima [4]. Then after, numerous nature inspired stochastic optimization technique like the hierarchal genetic algorithm (HGA) [5], hybrid taguchi genetic algorithm (HTGA) [6], taguchi immune algorithm (TIA) [7], real coded genetic algorithm (RCGA) [8], particle swarm optimization (PSO) [9], seeker optimization algorithm (SOA) [10], predator prey optimization (PPO) method [11], heuristic search method (HSM) [12], enhanced teaching-learning based optimization (ETLBO) technique [13] etc. have been developed and employed for optimal digital IIR filter designing. So, presently the development of new and efficient optimization algorithms for the designing of optimal digital IIR filters is very much in progress.

Therefore, in this paper, first the conventional CSO algorithm is used for the design of digital IIR LP filter. Further, an improvement in the form of opposition based learning is affixed with a chance of starting with better initial solutions by simultaneously checking the opposite solutions and the proposed improved version is implemented for the design of stable and optimal digital IIR LP filter. The multicriterion optimization approach is used as the design criterion that undertakes the minimization of magnitude approximation error and minimization of ripple magnitudes of the pass band as well as the stop band while satisfying the stability constraints that are imposed during the design process. A comparison of the results returned by the conventional CSO and improved CSO is carried out with other digital IIR filter design methods available in literature.

The remainder of this paper is organized as follows. Section 2 describes the digital IIR filter designing problem. The details of the mechanism for designing the digital IIR filter using cat swarm optimization algorithm and opposition based learning is described in section 3. Section 4 contains the proposed algorithm steps in detail. In section 5, the performance and statistical analysis of the proposed method has been carried out and the results obtained are compared with the design results in [3], [5], [6] and [15]. Finally, section 6 contains the concluding remarks and scope for future work.

2. Problem Statement

Traditionally, the design of digital IIR filter is generally realized by the following difference equation [1]:

$$y(n) = \sum_{i=0}^{M} x_i u(n-i) - \sum_{k=1}^{N} x_{N+k}\, y(n-k) \qquad (1)$$

where, M and N are the number of x_i and x_{N+k} filter coefficients, respectively, such that $N \geq M$. $u(n)$ and $y(n)$ are its input and output, respectively. An equivalent transfer function of digital IIR filter is expressed as follows:

$$H(z) = \frac{\sum_{i=0}^{M} x_i z^{-i}}{1 + \sum_{k=1}^{N} x_{N+k} z^{-k}} \qquad (2)$$

For designing of digital IIR filter the values of the filter coefficients x_i and x_{N+k}, which produce the desired response, are needed to be found out. In general the digital IIR filter is realized by cascading different first-order and second-order sections together. The transfer function of the cascaded digital IIR filter is denoted by $H(w,X)$, where X indicates the filter coefficients. The magnitude of $H(w,X)$ is denoted by $|H(w,X)|$. The basic structure of $H(w,X)$ can be stated as [3]:

$$H(w,X) = x_1 \prod_{i=1}^{M}\left(\frac{1+x_i e^{-jw}}{1+x_{2i+1}e^{-jw}}\right) \times \prod_{k=1}^{N}\left(\frac{1+x_l e^{-jw}+x_{l+1}e^{-2jw}}{1+x_{l+3}e^{-jw}+x_{l+4}e^{-2jw}}\right) \quad (3)$$

where, N and M denotes the number of filter coefficients of the first and second order sections, $l = 2M + 4(k-1)+2$ and vector $X = [x_1 x_2 \ldots\ldots x_D]^T$ denotes the filter coefficients of dimension D×1, such that, $D = 2M + 4N + 1$.

In the IIR filter design process, the coefficients are optimized so that the approximation error function for magnitude is minimized. The magnitude response is specified at K equally spaced discrete frequency points in pass-band and stop-band. The absolute error is denoted by $e(X)$ and is stated below:

$$e(X) = \sum_{k=0}^{K}\left|H_d(w_k) - |H(w_k,X)|\right| \quad (4)$$

where, $H_d(w_k)$ is the desired magnitude response of IIR filter and is given as:

$$H_d(w_k) = \begin{cases} 1 \text{ for } w_k \in passband \\ 0 \text{ for } w_k \in stopband \end{cases} \quad (5)$$

The ripple magnitudes of pass-band and stop-band are denoted by $\delta_p(x)$ and $\delta_s(x)$, respectively and are given as:

$$\delta_p(X) = max_{w_k}\{|H(w_k,X)|\} - min_{w_k}\{|H(w_k,X)|\};\ w_k \in passband, \quad (6)$$

$$\delta_s(X) = max_{w_k}\{|H(w_k,X)|\};\ w_k \in stopband \quad (7)$$

The design of stable digital IIR filter requires the inclusion of stability constraints. Therefore, the stability constraints obtained by using the jury method [14] on the coefficients of the digital IIR filter stated in Eq. (9.1) - Eq. (9.5), are used in the optimization process. The multivariable constrained optimization problem is then stated as:

$$Minimize\ f(x) = e(x) \quad (8)$$

Subject to the stability constraints:

$$1 + x_{2k+1} \geq 0(k = 1,2,\ldots,M) \quad (9.1)$$

$$1 - x_{2k+1} \geq 0(k = 1,2,\ldots,M) \quad (9.2)$$

$$1 - x_{l+3} \geq 0(l = 2M + 4(k-1)+2, k-1,2,\ldots N) \quad (9.3)$$

$$1 + x_{l+2} + x_{l+3} \geq 0(l = 2M + 4(k-1)+2, k-1,2,\ldots,N) \quad (9.4)$$

$$1 - x_{l+2} + x_{l+3} \geq 0(l = 2M + 4(k-1)+2, k-1,2,\ldots M) \quad (9.5)$$

Scalar objective constrained multivariable optimization problem is converted into scalar objective unconstrained multivariable optimization problem using exterior penalty function. Augmented objective function is defined as [15]:

$$A(x) = e(x) + r(P_{term}) \quad (10)$$

where,

$$P_{term} = \sum_{i=1}^{M}\langle 1+x_{2i+1}\rangle^2 + \sum_{i=1}^{M}\langle 1-x_{2i+1}\rangle^2 + \sum_{k=1}^{N}\langle 1-x_{l+3}\rangle^2 + \sum_{k=1}^{N}\langle 1+x_{l+2}+x_{l+3}\rangle^2 + \sum_{k=1}^{N}\langle 1-x_{l+2}+x_{l+3}\rangle^2 \quad (11)$$

r is a penalty term having a large value.

Bracket function for constraints given in Eqn. (9.1) and Eqn. (9.4) is stated below in Eqn. (12) and Eqn. (13) respectively:

$$\langle 1+x_{2i+1}\rangle = \begin{cases} 1+x_{2i+1}, if\left(1+x_{2i+1}\right)<0 \\ 0 \qquad ,if\left(1+x_{2i+1}\right)\geq 0 \end{cases} \tag{12}$$

$$\langle 1+x_{l+2}+x_{l+3}\rangle = \begin{cases} 1+x_{l+2}+x_{l+3}, if\left(1+x_{l+2}+x_{l+3}\right)<0 \\ 0 \qquad ,if\left(1+x_{l+2}+x_{l+3}\right)\geq 0 \end{cases} \tag{13}$$

Similarly, bracket functions for other constraints given by Eq. (9.2), Eq. (9.3) and Eq. (9.5) are undertaken. Initial feasible solutions are generated applying constraint handling method [15], in which filter coefficients are randomly perturbed till the satisfaction of constraints. During the run the penalty terms are perturbed to zero by applying random constraint handling.

3. Cat Swarm Optimization

CSO is a swarm intelligence based optimization algorithm which imitates the natural behaviour of cats. Cats have a strong interest towards moving objects and own excellent hunting skills. These two important behavioural traits of the cats are represented by seeking mode and tracing mode, respectively, which are mathematically modelled for solving complex optimization problems [16, 17].

3.1. Population Initialization

For implementing the CSO algorithm, initially the number of individuals in the population or cats that will take part in the optimization process is decided. Every individual/ cat in the population has a position which is made up of D-dimensions, velocities for each dimension, a fitness value according to the fitness function and a seeking/tracing flag. The position of the cat represents the candidate solution and the fitness value of each cat represents the accommodation of the cat to the fitness function. The seeking/tracing flag is used to identify whether the cat is in seeking mode or tracing mode. The population of cats within the solution search space is initialized as:

$$x_{id}^{\ t} = x_d^{\ min} + R\left(x_d^{\ max} - x_d^{\ min}\right)\left(d=1,2,...,D; i=1,2,...,T\right) \tag{14}$$

And the velocity for each dimension is mathematically given as:

$$v_{id}^{\ t} = v_d^{\ min} + R\left(x_d^{\ max} - v_d^{\ min}\right)\left(d=1,2,...,D; i=1,2,...,T\right) \tag{15}$$

where, x_{id}^{t} and v_{id}^{t} represents the position and velocity of the *ith* cat in *dth* dimension, respectively. And R is uniform random number between 0 and 1. The population may violate inequality constraints which are corrected by applying the random perturbation method.

3.2. Fitness Evaluation

The aim of the optimization process is to minimize the objective function. There is a possibility for the elements of parent/offspring to violate the constraint. Therefore, a penalty term is introduced and the objective function is penalized and changed to the following generalized form:

$$A_i\left(X_i\right) = e_i\left(X_i\right) + R\left(P_{term}\right)\left(i=1,2,...,T\right) \tag{16}$$

where, $X_i = [x_{i1}, x_{i2}, ..., x_{iD}]^T$ and penalty factor is given by Eqn. (11). The value increases with the progress of the algorithm.

3.3. Oppositional Learning

The opposition based learning strategy helps CSO algorithm to take a start with some initial random solutions which are improved over time by moving towards some optimal solution. The computational time of an algorithm is an important parameter that is related to the remoteness of the initial guesses from the optimal solution. This can be improved by starting with a better solution while checking the opposite solution in the search space. The guess or its opposite guess is chosen as initial solutions. A guess is farther from the solution than its opposite guess with 50% probability [18]. Therefore, starting with better guesses adjudged by its objective function has the ability to increase the convergence speed. During the run, the same approach can be applied not only to initial solutions but also continuously to each solution in the current population to reach a final optimal solution. This can be mathematically expressed as:

$$x_{i+T,d} = x_d{}^l + x_d{}^u - x_{id}\, t\left(d = 1,2,...,D;\, i = 1,2,...,T\right) \quad (17)$$

where, $x_d{}^l$ and $x_d{}^u$ are lower and upper limits of filter coefficients, respectively and are expressed asfollows:

$$x_d{}^l = \begin{cases} x_d{}^{min} & ;t = 1 \\ min\{x_{id}\,; i = 1,2,\ldots,T\}; & t > 1 \end{cases} \quad (18)$$

$$x_d{}^u = \begin{cases} x_d{}^{max} & ;t = 1 \\ max\{x_{id}\,; i = 1,2,\ldots,T\}; & t > 1 \end{cases} \quad (19)$$

3.4. Seeking mode

A user predefined value, MR called the mixture ratio is used to set the seeking/tracing flag, which decides the number of cats that would randomly be moved into the seeking mode and the tracing mode [19, 20].
The seeking mode corresponds to the global search in the CSO optimization algorithm. This mode imitates the observant behaviour of cats by creating copies of the current solution. Each copy then tries to improve the given solution through the exploitation process. After all copies have finished exploiting the current solution, a new solution which would replace the current solution is selected. This new solution would represent the new position on which the cat has to move. Seeking mode incorporates four important parameters namely Memory Seeking Pool (MSP), Seeking Range of Dimension (SRD), Counts of Dimension to Change (CDC) and Self position consideration (SPC). For a real cat, MSP is defined as the size of seeking memory for each cat indicating the points sought by each cat. SRD the dictates the mutative ration for the selected dimensions. If a dimension is selected to mutate, the maximum difference between the new value and the old value cannot be out of the range defined by SRD. CDC indicates how many dimensions will be varied and SPS is a Boolean variable which decides whether the point on which the cat is already standing is a point, one of the candidates to move to. The seeking mode involves the generation of t copies of the present position of cat i, where t = MSP. If the value of SPC is true, let t = (MSP − 1), then retain the present position as one of the candidates. For each copy, according to CDC, randomly plus or minus SRD percents the present values and replace the old ones according to the following mathematical equations:

$$X_{id}{}^c = X_{id} + cnvRS_{rd}\, X_{id}\left(d = 1,2,...,D;\, i = 1,2,...,T\right) \quad (20)$$

$$X_{id}{}^c = X_{id} - cnvRS_{rd}\, X_{id}\left(d = 1,2,...,D;\, i = 1,2,....,T\right) \quad (21)$$

At the end of the seeking mode, the fitness of all copies is evaluated and from t copies the candidate with best fitness is selected and placed at the position of *ith*cat.

3.5. Tracing mode

The tracing mode corresponds to the local search technique where the rapid chase of the cat for its pray is mathematically modelled as a large change in its position. Then the position and the velocity of *ith* cat in the D-dimensional space are mathematically expressed as follows:

$$X_i = \left[x_{i1}, x_{i2}, ..., x_{iD}\right]^T \text{ and,} \quad (22)$$

$$V_i = \left[v_{i1}, v_{i2}, ..., v_{iD}\right]^T \quad (23)$$

The global best position of the cat is represented by X_g, where $X_g = \left[x_{g1}, x_{g2}, ..., x_{gD}\right]^T$. In the tracing mode the velocity and the position of the *ith* cat are updated using the following equations:

$$V_{id}{}^n = wV_{id} + CR\left(X_{gd} - X_{id}\right)\left(d = 1,2,...,D;\, i = 1,2,...,T\right) \text{and,} \quad (24)$$

$$X_{id} = X_{id} + V_{id}^{\,n} \left(d = 1,2,\ldots,D; i = 1,2,\ldots,T \right) \quad (25)$$

where, w represents the inertia weight, C is the acceleration constant and R is a uniform random number distributed in the range [0, 1].

4. Developed Algorithm

The digital IIR LP filter designing is carried out using the CSO algorithm with the opposition based learning technique. Here, the developed algorithm tries to have an optimal IIR LP filter structure while satisfying the stability constraints that are imposed during the designing. The implementation of the proposed algorithm for digital IIR LP filter design is explained step by step as follows:

The Main Procedure of CSO Algorithm

1. Initialize the algorithm parameters like number of cats i.e. the population size (NC), maximum iteration (ITMAX), mixture ratio (MR), memory seeking pool (MSP), seeking range of dimension (SRD), counts of dimension to change (CDC), self position consideration (SPC), C1, x^{max} and x^{min}.
2. Set t=0; generate an array of (D×T) size of uniform random numbers.

FOR d=1 to D

FOR i=1 to T

3. Randomly initialize the position of cats in D-dimensional space for the population, i.e. $x_{id}^{\,0}$, using Eqn. (14).
4. Randomly initialize the velocity for cats, i.e. $v_{id}^{\,0}$, using Eqn. (15).
5. Compute the augmented objective function $A_i(x_{id}^{\,0})$, using Eqn. (16).
6. Generate the initial population of individuals using opposition, Eqn. (17).
7. Compute the augmented objective function $A_{i+T}(x_{i+T,d}^{\,0})$, using Eqn. (16).
8. Compare $A_i(x_{id}^{\,0})$ and $A_{i+T}(x_{i+T,d}^{\,0})$.

END FOR

END FOR

9. Arrange Ai in ascending order and select first T cats out of 2T cats in the swarm.
10. Select best member with highest fitness out of T cats as $A_b^{\,0}$ and select the corresponding position as $X_{b_{id}}^{\,0}$.

WHILE (T≤ T^{max}) DO

11. Increment the iteration count, t=t+1.

IF (seeking/tracing flag=1) THEN

12. Apply seeking mode steps given in Eqn. (20) and Eqn. (21).

ELSE

13. Apply tracing mode steps given in Eqn. (24) and Eqn. (25).

ENDIF

14. Select best member Abest and corresponding position as (Xid)best.

19. IF ($A_{best} < A_b^{\,0}$) THEN

$A_b^{\,0}$=A_{best};

$X_{b_{id}}^{\,0} = G_{b_{id}}$

ENDIF

ENDDO

5. Design Results and Comparison

5.1. Lower order digital IIR filter design

The design of digital IIR LP filter has been carried out and the filter coefficients have been evaluated using improved cat swarm optimization with oppositional learning. For the purpose of comparison, the lowest order of the digital IIR LP filter is set exactly same as that set by Tang *et al.* [3] and Tsai *et al.* [5] i.e. the order is set equal to 3. For designing the digital IIR LP filter, 200 equally spaced points are set within the frequency domain [0, π]. The objective of the optimization problem is to minimize the magnitude approximation error and ripple magnitudes of both the pass-band and the stop-band, subject to the stability constraints given by Eq. (9.1) - Eq. (9.5) under the prescribed design conditions stated in Table 1. The control parameters settings for the improved CSO algorithm are listed in Table 2. The final filter model obtained for the LP filter is given in Eq. (26) and the frequency response and pole-zero plot are represented in Fig. 1 and Fig. 2, respectively. The results obtained for the LP filter by implementing CSO as well as the improved CSO algorithm are summarized in Table 3, where the comparison of the obtained results is carried out with the design results given by other methods like HGA [3], HTGA [5], TIA [6], and RCGA [15]. It is observed that the improved CSO algorithm is capable of producing results that are superior as compared to the results given by the conventional CSO and other digital IIR filter design algorithms.

Table 1. Prescribed design conditions for LP filter

Maximum value of $\left\|H\left(w_i, x\right)\right\|$	1
Pass band	$0 \le w \le 0.2\pi$
Stop band	$0.3\pi \le w \le \pi$

Table 2. Values of control parameters for LP filter

Parameter	Notation	Low-pass filter
Number of cats	NC	100
Mixture ratio	XMR	0.90
Maximum number of iterations	ITMAX	200
Memory seeking pool	MSP	5
Seeking range of dimension	SRD	0.85
Counts of dimension to change	CDC	0.20

$$H_{LP(z)} = 0.0328\frac{\left(z+0.9093\right)\left(z^2-0.2332z-1.3874\right)}{\left(z-0.6690\right)\left(z^2-1.3874z+0.7371\right)} \tag{26}$$

Table 3. Design results for LP filter

Method	Magnitude error	Filter order	Pass band ripples	Stop band ripples
HGA[5]	4.3395	3	$0.8870 \le \|H(e^{jw})\| \le 1.0090$ (0.1139)	$\|H(e^{jw})\| \le 0.1802$ (0.1802)
HTGA[6]	4.2511	3	$0.9000 \le \|H(e^{jw})\| \le 1.0000$ (0.0996)	$\|H(e^{jw})\| \le 0.1247$ (0.1247)

TIA[7]	3.8157	3	0.8914≤|H(e^jw)|≤1.0000 (0.1086)	|H(e^jw)|≤0.1638 (0.1638)
RCGA[8]	4.0095	3	0.9355≤|H(e^jw)|≤1.0160 (0.0825)	|H(e^jw)|≤0.1510 (0.1510)
HSM [12]	4.1145	3	0.9246≤|H(e^jw)|≤1.0110 (0.0871)	|H(e^jw)|≤0.1238 (0.1238)
ETLBO [13]	4.0482	3	0.9117≤|H(e^jw)|≤1.0060 (0.0497)	|H(e^jw)|≤0.1217 (0.1217)
CSO	3.7759	3	0.9376≤|H(e^jw)|≤1.021 (0.0835)	|H(e^jw)|≤0.1567 (0.1567)
Opposition aided CSO	**3.6647**	**3**	**0.9461≤|H(e^jw)|≤1.0378 (0.0847)**	**|H(e^jw)|≤0.1661 (0.1191)**

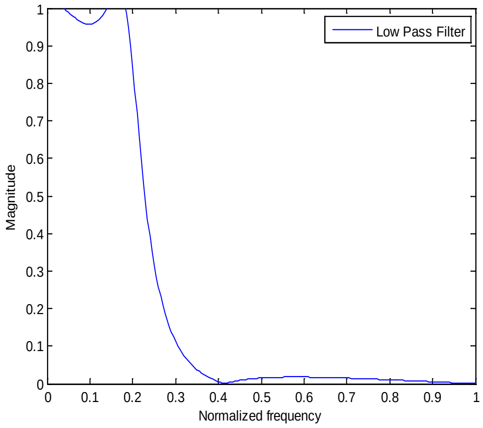


Fig. 1. Magnitude response of the LP filter

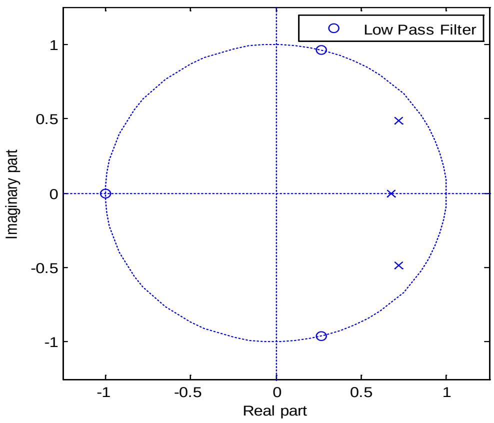


Fig. 2. Pole-zero plot of the LP filter

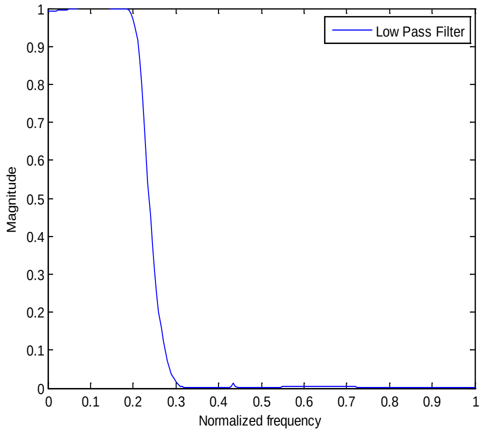
5.2. Higher order digital IIR filter design

Further, the improved CSO algorithm with oppositional learning has been employed for designing the higher order digital IIR filter LP filter. Like the lower order design, 200 equally spaced points are set within the frequency domain [0, π]. The order of the digital IIR filter is given as *M+2N*; where *M* and *N* denotes the number of filter coefficients. By varying the values of M and N, the value of the order has been varied ranging from 1 to 15. Here, the objective of the optimization problem is same i.e. to minimize the absolute error (i.e., L_1-norm) of magnitude response subject to the stability constraints given by Eq. (9.1) - Eq. (9.5) under the prescribed design conditions given in Table 2.

For higher order LP filter design, the maximum number of iterations for the proposed algorithm has been set to 250. An initial population of 100 cats is considered with a mixture ratio (*MR*) of 0.85. The values of *MSP*, *SRD* and *CDC* are taken as 5, 0.20 and 0.80, respectively. The proposed algorithm shows the capability to design a stable and optimal IIR LP filter with values of M and N equal to 1 and 4, respectively i.e. the order of the filter is 9. This designed filter with an order of 9 showed better magnitude approximation error over all other orders. The magnitude approximation error and the pass-band and stop-band ripple magnitudes for the higher order LP are summed up in Table 4. The values of the optimized filter coefficients obtained for the higher order digital IIR LP filter are given in Table 5 and the frequency response and pole-zero diagrams are given in Fig. 3 and Fig. 4, respectively. As all the poles lie inside the unit circle, this shows that the designed high order LP filter is stable and it strictly follows the stability constraints that are imposed during its designing.

Table 4. Design results for higher order LP filter

Magnitude Error	Pass band ripples	Stop band ripples
1.6362	0.9598≤|H(e^jw)|≤ 1.0105 (.0507)	|H(e^jw)|≤ 0.0149 (0.0149)



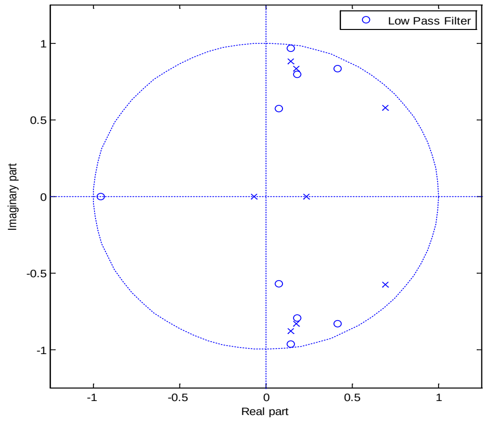


Fig. 3. Frequency response of order 9 LP filter **Fig. 4.** Pole-zero plot of order 9 LP filter

Table 5. Coefficients of higher order digital IIR LP filter model

i	a_i	b_i	p_i	q_i	r_i	s_i
1	0.92007	-0.31627	-0.79761	0.79144	-0.21356	0.81255
2			-0.40367	0.69651	-0.40265	0.73159
3			-0.19377	0.37456	-1.26411	0.80873
4			-0.31746	0.89345	0.12919	0.71165

6. Robustness and Statistical Analysis

Robustness is a significant feature for evaluating the performance of any evolutionary algorithm. The CSO algorithm takes a start with random initialization of the population of cats, thus making randomness an inherent feature of CSO. Therefore, the robustness of CSO algorithm to achieve global optimum design solution for order 3 and order 9 LP filter is determined by having 100 independent trial runs with random seed numbers for each case. The variations in the value of overall objective function have been observed. The maximum value, minimum value, average value and standard deviation in the value of overall objective function are calculated and given in Table 6. From Table 6, it can be observed that for both the cases, the value of standard deviation is very small which indicates that the CSO algorithm possesses outstandingly strong robustness.

To further validate the obtained results and to confirm the effectiveness of the developed algorithm for digital IIR LP filter design, a non-parametric statistical test called the Wilcoxon's signed rank test for single sample is used. This test is conducted on the results obtained by the developed algorithm for the LP filter with a significant level of $\alpha = 0.10$ by comparing them with the results provided by other existing algorithms. Firstly, the sum of positive ranks (R+) and the sum of negative ranks (R-) is calculated and then the p-value is determined in each case. The Wilcoxon's signed rank test (Table 7) depicts that the results of the developed algorithm are significantly better then the HGA, HTGA, TIA and RCGA algorithms as the p-value is less than 0.10 in all the cases and facilitates the designing of not only stable but optimal digital IIR LP filter.

Table 6. Maximum, minimum, average and standard deviation of magnitude error for LP filter for lower and higher order values

Order	Maximum magnitude Error	Minimum magnitude Error	Average magnitude Error	Standard Deviation of magnitude Error
3	5.6923	3.6375	4.9601	0.0694
9	3.4901	1.4147	2.0149	0.4392

Table 7. Statistical analysis results based on Wilcoxon's signed rank test for lower order LP filter

Performance	α	R^+	R^-	*p*-value
Magnitude approximation error	0.10	0	10	0.033945
Pass-band performance	0.10	1	9	0.072064
Stop-band performance	0.10	1	9	0.072064

7. Conclusion

With the use of digital IIR filters in almost all the engineering application areas, the designing of digital IIR filters is increasingly gaining interest. This paper proposes a population based stochastic optimization algorithm i.e. CSO to design the optimal and stable digital IIR LP filter. CSO possesses merits like robustness and local as well global search abilities and thus is capable of returning a global optimal solution which is not possible in some conventional optimization algorithms. For further improving the solution and to take a start with better solution set, opposition based learning strategy is also incorporated. While moving towards the optimal solution, oppositional learning method simultaneously checks the opposite solutions, thus extensive exploration and exploitation of the entire solution search space is carried out. The proposed approach is executed to solve the multi criterion optimization problem of designing digital IIR LP filter. The experimental results show that the results obtained by CSO algorithm in terms of magnitude response error and ripple magnitudes of pass band and the stop band are better than the results given in [3], [5], [6] and [15] and is very much feasible for the designing of digital IIR LP filter when the multi criteria, complicated constraints, and design requirements are involved. Further, the pole-zero plot of the digital IIR LP filter depicts the stability of the designed filter as all the poles lies inside the unit circle. Also, developed algorithm performs well for designing the LP filter having higher order. For future research, parameter tuning is still a potential area. Also, the proposed method can be used to design higher order multi dimensional filters, adaptive filters and filter banks.

References

[1] J.G. Proakis, *Digital Signal Processing*, Prentice-Hall International. Inc., New Jersey. 2010.

[2] E. C. Ifeachor and B. W. Jervis, *Digital Signal Processing, APractical Approach*, 2nd edition, Pearson Education, Singapore, 2003.

[3] S. K. Mitra and J. F. Kaiser, *Handbook for Digital Signal Processing*, Wiley, New York, 1993.

[4] G. Panda, P. M. Pradhan and B. Majhi, "IIR System Identification Using Cat Swarm Optimization," *Expert Systems with Applications*, vol. 38, no. 10, pp. 12671-12683, September 2011.

[5] K. S. Tang, K. F. Man, S. Kwong and Z. F. Liu, "Design and Optimization of IIR Filter Structure using Hierarchical Genetic Algorithms," *IEEE Transaction on Industrial Electronics*, vol. 45, no. 3, pp. 481–487, June 1998.

[6] J.-T. Tsai, J.-H. Chou and T.-K. Liu, "Optimal Design of Digital IIR filters by using Hybrid Taguchi Genetic Algorithm," *IEEE Transactions on Industrial Electronics*, vol. 53, no. 3, pp. 867–879, 2006.

[7] J.-T. Tsai and J.- H. Chou, "Optimal Design of Digital IIR Filters using an Improved Immune Algorithm," *IEEE transactions on Signal Processing*, vol. 54, no. 12, pp. 4582–4596, 2006.
[8] R. Kaur, M. S. Patterh and J. S. Dhillon, "Real Coded Genetic Algorithm for Design of IIR Digital Filter with Conflicting Objectives," *International Journal of Applied Mathematics and Information Sciences*, vol. 8, no. 5, pp. 2635-2644, 2014.
[9] S. K. Saha, R. Kar, D. Mandal and S. P. Ghoshal, " Novel Particle Swarm Optimization for Low Pass FIR Filter Design, *WSEAS Transactions on Signal Processing*, vol. 8, pp. 111-120, 2012.
[10] C. Dai and W. Chen, "Seeker optimization algorithm for digital IIR filter design," *IEEE Transactions on Industrial Electronics*, vol. 57, pp. 1710-1718, 2010.
[11] B. Singh and J. S. Dhillon, "Predator Prey Optimization Method for the Design of IIR Filter," *WSEAS Transactions on Signal Processing*, vol. 9, pp. 51-62, 2013.
[12] R. Kaur and J. S. Dhillon, " Heuristic Search Method for the Design of IIR filter, *WSEAS Transactions on Signal Processing*, vol. 8, pp. 121-134, 2012.
[13] D. Singh and J. S. Dhillon, " Design of Optimal IIR Digital Filter using Teaching-Learning based Optimization Technique, *WSEAS Transactions on Advances in Engineering Education*, vol. 12, pp. 9-18, 2015.
[14] I. Jury, *Theory and Application of the Z-Transform Method*, New York: Wiley, 1964.
[15] A. Jiang and H. K. Kwan, "IIR Digital Filter Design with New Stability Constraint based on Argument Principle," *IEEE Transactions on Circuit and Systems-I*, vol. 56, no. 3, pp. 583–593, March 2009.
[16] P.-W. Tsai, J.-S. Pan, S.-M. Chen and B.-Y. Lio, "Enhanced Parallel Cat Swarm Optimization based on the Taguchi method," *Expert Systems with Applications*, vol. 39, no. 10, pp. 6309-6319, 2012.
[17] P. M. Mohan and G. Panda, "Solving Multiobjective Problems using Cat Swarm Optimization," *Expert Systems with Applications*, vol. 39, no. 10, pp. 2956-2964, 2012.
[18] Rahnamayan, H. R. Tizhoosh and M.A. Salama, "Opposition based Differential Evolution," *IEEE Transactions on Evolutionary Computations*, vol. 12, no.1, pp 64-79, February 2008.
[19] S. C. Chu and P. W. Tsai, "Computational Intelligence based on the Behavior of Cat," *International Journal of Innovative computing, Information and Control*, vol. 3, no. 1, pp.163-173, 2007.
[20] Z.-H. Wang, C.-C. Chang and M.-C. Li, "Optimizing Least-Significant-Bit Substitution using Cat Swarm Optimization Strategy," *Information Sciences*, vol. 192, no., pp. 98-108, 2012.