Particle Swarm Optimization Algorithm for Designing BP and BS IIR Digital Filter

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Abstract. In this paper band pass (BP) and band stop (BS) infinite impulse response (IIR) filter is designed using particle swarm optimization (PSO) algorithm.The magnitude response of the IIR filter is approximated using L_1-approximation error criterion. The PSO algorithm is aoptimization technique inspired by genetics and natural evolution.The method enhances the search capability and provides a fast convergences for calculating the optimal filter coefficients. The filter designed based on L_1-approximation error possesses flat passbands and stopbands while keeping the transition band comparable to that of the least square design. A comparison has been made with other design techniques, demonstrating that PSO with enhanced diversity and convergence gives better or at least comparable results or designing digital IIR filters than the existing genetic algorithm based methods.

Keywords: Digital IIR filters, Particle Swarm Optimization, L_1-approximation error, magnitude response, Stability

1. Introduction

Filtering is a process with which a frequency spectrum of a signal can be adjusted according to desired specifications given by the designer. Digital filter are used in number of application like speech recognition, image enhancement, radar processing, secure communication and biomedical engineering, so great attention is required for efficient designing of digital filter. Digital filters are of two types; finite impulse response (FIR) and infinite impulse response (IIR) filter. IIR filters are more efficient than FIR filters because, for a given frequency response, they require fewer delay elements, adders, and multipliers. An IIR filter can give a sharper cutoff than an FIR filter of the same order because both poles and zeros are present. However, a causal IIR filter cannot achieve exactly linear phase but the FIR filter can [1].The digital IIR filter design is essentially a multiparameter and multicriterion optimization problem with multiple local optima, the conventional gradient-based design methods may easily get stuck in the local minima of error surface. So, it is very necessary to develop efficient optimization algorithms to deal with digital IIR filter design problems. Some researchers have developed design methods based on modern heuristics optimization algorithms such as genetic algorithms [2-6], seeker-optimization-algorithm-based evolutionary method [7], simulated annealing [8], tabu search [9], ant colony optimization [10], hybrid taguchi genetic algorithm (HTGA) [11] ,immune algorithm (TIA) [12] and many more.

PSO is a global optimisation technique which is very simple to implement and converge quickly. Due to the efficiency and capability of PSO in the minimization of multimodal functions with numerous local and global minima, the PSO and its variants has been verified by various researchers in digital filter design problems [13-18]. In this paper PSO is applied for magnitude approximation of band-pass (BP) and band-stop (BS) IIR digital filter.The values of the filter coefficients are optimized with PSO approach to minimize magnitude

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response error with L_{1}- error criterion. The paper is arranged as follows. Section 2 describes the IIR filter design problem statement. The PSO algorithm for designing the optimal digital IIR filters is described in Section 3. In Section 4, the performance of the proposed method has been evaluated and achieved results are compared with the design results by Tsai and Chou [12] and Tang et al. [4] for the BP and BS filters. Finally, the conclusions and discussions are given in Section 5.

2. Problem Formulation

The objective function in the designing of IIR filter is to minimize difference between the actual and the specified magnitude response with respect to the transfer function coefficients. The output of IIR filter depends on past outputs and present inputs. The rational transfer function of IIR in cascading form by cascading first and second order sections is [5]:

$$H(\omega, x) = A \prod_{i=1}^{M} \frac{1 + a_{1i} e^{\;j\omega}}{1 + b_{1i} e^{\;j\omega}} \times \prod_{k=1}^{N} \frac{1 + c_{1k} e^{\;j\omega} + c_{2k} e^{\;2j\omega}}{1 + d_{1k} e^{\;j\omega} + d_{2k} e^{\;2j\omega}} \quad (1)$$

Where

$x = [a_{11}, b_{11}, ..., a_{1M}, b_{1M}, c_{11}, c_{21}, d_{11}, d_{21}, ..., c_{1N}, c_{2N}, d_{1N}, d_{2N}, A]^T$. The Vector x denotes the filter coefficients of dimension W×1 with W = 2M + 4N + 1 and ω represents the discrete frequency .The design aim of the paper is to minimize magnitude error function in L_1-norm for LP and HP by optimizing the filter coefficient vector x. The magnitude response is specified at V equally spaced discrete frequency points in pass-band and stop-band. The L_p-norm approximation error for the magnitude response is defined as [12].

$$e(x) = \sum_{i=0}^{V} \left| H_d(\omega_i) \quad |H(\omega_i, x)| \right|^{p \; 1/p} \quad (2)$$

For p=1, the magnitude response error denotes the L_1-norm error and is defined as given below:

$$e(x) = \sum_{i=0}^{V} \left| H_d(\omega_i) \quad |H(\omega_i, x)| \right| \quad (3)$$

Desired magnitude response $H_d(\omega_i)$ of IIR filter is given as:

$$H_d(\omega_i) = \begin{cases} 1, & for \;\; \omega_i \in passband \\ 0, & for \;\; \omega_i \in stopband \end{cases} \quad (4)$$

Minimize $J(x) = e(x)$ (5)

Subject to: the stability constraints:

$1 + b_{1i} \geq 0 \;\; (i = 1, 2, ..., M)$ (5a)

$1 \quad b_{1i} \geq 0 \;\; (i = 1, 2, ..., M)$ (5b)

$1 \quad d_{2k} \geq 0 \; (k = 1, 2, ..., N)$ (5c)

$1 + d_{1k} + d_{2k} \geq 0 \; (k = 1, 2, ..., N)$ (5d)

$1 \quad d_{1k} + d_{2k} \geq 0 \; (k = 1, 2, ..., N)$ (5e)

For stable IIR digital filter, all the poles should lie inside the unit circle. Therefore, the stability constraints given by Eq. (5a) to Eq. (5e) which are obtained by using the Jury method [19], are imposed in the design procedure on the coefficients of the digital IIR filter..

3. Design Methodology

PSO is an efficient global population-based optimization algorithm that manoeuvres and exploits a population of individuals to probe promising regions of the search space. In this context, the population is called a swarm and the individuals are called particles. In every iteration, each particle is updated by two best values. The first one is the best solution it has achieved so far. Second is the best value obtained so far by any particle in the population [20].

Assume NV is swarm size and a swarm consists of V particles. The jth particle is represented as $x_i = \lfloor x_{i1}, x_{i2}, ...x_{iV} \rfloor$. The rate of the velocity for this particle is represented as $v_i = \lfloor v_{i1}, v_{i2}, ..., v_{iV} \rfloor$. The best previous position of each particle is recorded and represented as $xb_i = \lfloor xb_{i1}, xb_{i2}, ..., xb_{iV} \rfloor$. The index of the best particle among all the particles in the group is represented by the $\lfloor G_1, G_2, ..., G_V \rfloor$. The modified velocity and position of each particle can be calculated using the current velocity and the distance from xb_{ij} to G_j as shown in the following formula:

$$v_{ij}^{r+1} = C_{fk} \times \left\{ w \times v_{ij}^r + C_1 \times R_1 \times (xb_{ij}^r - x_{ij}^r) + C_2 \times R_2 \times (G_j^r - x_{ij}^r) \right\} \quad (i = 1,2,....., NV;\ j = 1,2,....., V) \tag{6}$$

$$x_{ij}^{r+1} = x_{ij}^r + v_{ij}^{r+1} \quad (i = 1, 2, ..., NV;\ j = 1, 2, ..., V) \tag{7}$$

where

NV is size of a swarm.

V is the number of particles in a swarm.

R is the pointer of iterations. w is the inertia weight factor.

C_1 and C_2 are the acceleration constants.

R_1 and R_2 are uniform random values in the range [0,1].

v_{ij}^r is the velocity of jth member of ith particle at rth iteration, $V_j^{min} \leq v_{ij}^r \leq V_j^{max}$.

x_{ij}^r is the current position of jth member of ith particle at rth iteration.

C_{fk} is the constriction factor.

While implementing PSO the following considerations have been taken into account to facilitate the convergence and prevent an outburst of the swarm:

A. Maximum Velocity

The velocity of the particle is a stochastic variable and is, therefore, subject to create an uncontrolled trajectory, making the particle follow wider cycles in the problem space [21,22]. In order to control these oscillations, upper (V_j^{max}) and lower (V_j^{min}) limits are defined for the velocity.

B. Acceleration Constants

The acceleration constants C_1 and C_2 determine the effect of personal best xb_{ij}^r and global best G_j^r positions. Low values limit the movement of the particles and high values result in abrupt movement toward, or past, target regions. Hence the acceleration constants C_1 and C_2 are set to be 2.0 according to past experiences.

C Constriction factor / Inertia Weight with chaotic sequence

Clerc and Kennedy [23] were first to introduce constriction coefficient and is defined as follows:

$$C_{fk} = \frac{2}{\left| 2 - \varphi - \sqrt{\varphi^2 - 4\varphi} \right|}, \quad C_1 + C_2 = \varphi > 4.0 \tag{8}$$

In general, the constriction factor dampens the movement, once the particle is focused on the best point in an optimal region thus helping the convergence of the particle.

This parameter controls the exploration of the search space. Initially higher value is selected to allow the particles to move freely in order to find the global optimum neighbourhood fast. Once the optimal region is found, the value of the inertia weight is reduced to narrow the search. As originally developed by Shi and Eberhart[25], w often decreases linearly from 0.9 to 0.4 during a run. In general the inertia weight w is set according to the following equation:

$$w_i = w^{max} - \frac{w^{max} - w^{min}}{IT^{max}} \times IT \tag{9}$$

where
IT^{max} is the maximum number of iterations.
IT is the current number of iterations.

D. Initialization of swarm

Random search is applied to initialize each element of swarm matrix. Global search is applied to explore the starting point and then the starting point is perturbed in local search space to record the best starting point. The search process is started by initializing the variable x_{ij}^{t} using equation 7 which is used to calculate objective function equation 5.

$$x_{ij}^{0} = x_j^{min} + rand()(x_j^{max} \quad x_j^{min}) \qquad (i = 1, 2, ..., NV ; j = 1, 2, ..., V) \tag{10}$$

where rand is a uniform random generated number having value between 0 and 1, V is number of particles, NV is the size of swarm and x_i^{max} and x_i^{min} represents the maximum and minimum limits of i[th] decision variable of vector X.

4. Design Examples and Comparisons

For the purpose of comparison, the lowest order and design conditions of the digital IIR filter are set exactly the same as that given by Tang et al. in [4] for the BP and BS filters. The objective of designing the digital IIR filters is to minimize the objective function given by Eq. (5) with the stability constraints stated by Eq. (5a) to Eq. (5e) under the prescribed design conditions given in Table I. The examples of the IIR filters considered by Tsai and Chou [12] and Tang et al. [4] are considered to test and compare the performance of proposed PSO approach.
From the evaluated results with the proposed method in Tables 1-2 and Figs.1 - 2, it can be observed that, for the BP, and BS filters, the proposed PSO approach gives the smaller L_1-norm approximation errors and the better magnitude performances in both pass-band and stop-band than the genetic algorithm based method given by Tsai and Chou [12] and Tang et al. [4] respectively. The designed IIR filter models obtained by the PSO approach are given below**.**

$$H_{BP}(z) = 0.028636 \; \frac{(z^2 - 0.028142\,z - 0.905718\,)(z^2 - 0.000412z - 0.909829)}{(z^2 - \;\; 0.002875\,z + 0.541411\,)(z^2 - \;\; 0.629501\,z + 0.768064)} \times \frac{(z^2 + 0.001064 - 1.013981\,)}{(z^2 + 0.641416z + 0.773995\,)} \tag{11}$$

$$H_{BS} = 0.430690 \frac{(z^2 + 0.429296z + 0.989379\,)(z^2 - \;\; 0.407464z + 0.985270\,)}{(z^2 + 0.841328z + 0.543611\,)(z^2 - \;\; 0.821933z + 0.515733\,)} \tag{12}$$

Table 1. Design results for band-pass filter

Method	L_1-norm error	Pass-band performance (Ripple magnitude)	Stop-band performance (Ripple magnitude)
PSO Approach	1.3099	$0.9844 \le \lvert H(e^{j\omega}) \rvert \le 1.004$ (0. 0205)	$\lvert H(e^{j\omega}) \rvert \le 0.0546$ (0.0546)
TIA Approach[17]	1.5204	$0.9681 \le \lvert H(e^{j\omega}) \rvert \le 1.000$ (0.0319)	$\lvert H(e^{j\omega}) \rvert \le 0.0679$ (0.0679)
Method of Tang et al. [5]	5.2165	$0.8956 \le \lvert H(e^{j\omega}) \rvert \le 1.000$ (0.1044)	$\lvert H(e^{j\omega}) \rvert \le 0.1772$ (0.1772)

Table 2. Design results for band-stop filter

Method	L_1-norm error	Pass-band performance (Ripple magnitude)	Stop-band performance (Ripple magnitude)
PSO Approach	3.4015	$0.9485 \le \lvert H(e^{j\omega})\rvert \le 1.014$ (0.0663)	$\lvert H(e^{j\omega})\rvert \le 0.1152$ (0.1152)
TIA Approach[17]	3.4750	$0.9259 \le \lvert H(e^{j\omega})\rvert \le 1.000$ (0.0741)	$\lvert H(e^{j\omega})\rvert \le 0.1178$ (0.1278)
Method of Tang et al. [5]	6.6072	$0.8920 \le \lvert H(e^{j\omega})\rvert \le 1.000$ (0.1080)	$\lvert H(e^{j\omega})\rvert \le 0.1726$ (0.1726)

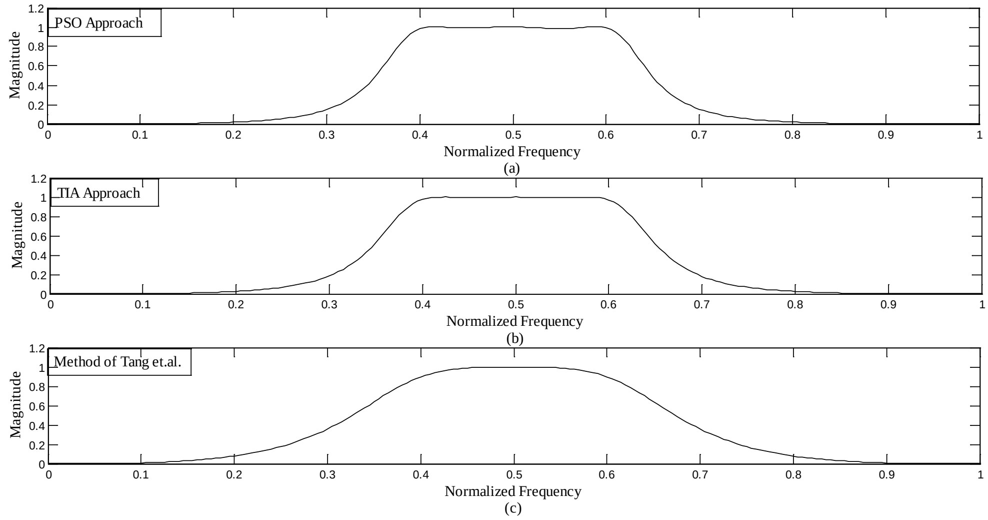

Fig. 1. Frequency responses of band pass filter using the PSO approach and the method given in [12] and [4], respectively

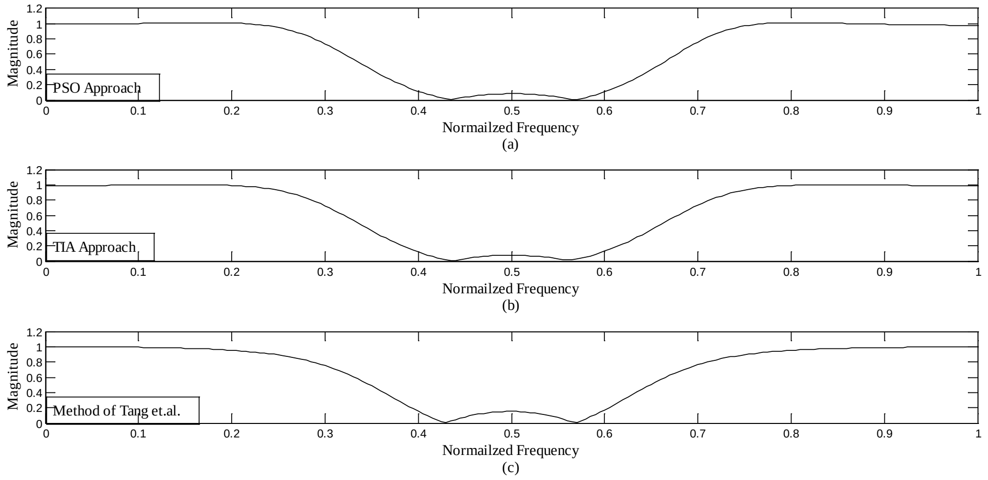

Fig. 2. Frequency responses of band stop filter using the PSO approach and the method given in [12] and [4], respectively

5. Conclusion

This paper proposes a PSO method for the design of BP and BS digital IIR filters based on L_1-norm approximation error for magnitude response approximation. In order to strengthen the PSO approach various additional factors have been taken care of like: random initialization of swarm, maximum velocity, constriction factor and inertia weight. As shown through simulation results, PSO algorithm gives better performance as compared to GAs in terms of magnitude response and attenuation in passband and stopband. The main advantages of the PSO algorithm are summarized as; simplicity in implementation, robust against local minimum problem and has a fast convergence speed.

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