Review Paper on Generalized Rician Fading

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Abstract. The objective of this paper is to review the progress made so far in Generalized Rician Fading Channel. Generalized Rician fading channel is a recent fading channel. It was introduced in 2001. It has n degrees of freedom. For n=2 generalized Rician fading channel becomes Rician fading channel. For n=2m it becomes Nakagami-m fading channel. So it unifies Rician fading channel and Nakagami-m fading channel. Generalized Rician fading channel gives better performance than other fading channels. Error probability analysis for different modulation schemes using various diversity schemes have been done using coherent or non coherent detection in uncorrelated or correlated rician fading channel.

Keywords: Rayleigh fading, Rician fading, Nakagami fading, Generalized Rician fading, Bit Error Rate.

1. Introduction

Wireless communications is by any measure, the fastest growing segment of the communications industry. As such it has captured the attention of the media and the imagination of the public. Cellular systems have experienced exponential growth over the last decade and there are currently around two billion users worldwide [1]. Fading limits the performance of wireless systems. The term fading or small-scale fading means rapid fluctuations of the amplitudes, phases or multipath delays of a radio signal over a short period or short travel distance. This might be so severe that large scale radio propagation loss effects might be ignored. In principle, the following are the main multipath effects: 1. Rapid changes in signal strength over a small travel distance or time interval. 2. Random frequency modulation due to varying Doppler shifts on different multipath signals. 3. Time dispersion or echoes caused by multipath propagation delays [2].
If X_1 and X_2 are two independent and identically distributed (iid) Gaussian random variables each distributed according to $N(0, \sigma^2)$ that is mean is 0 and varience is σ^2 then $x = \sqrt{X_1^2 + X_2^2}$ PDF of a Rayleigh random variable is given by

$$p_x = \frac{x}{\sigma^2} e^{\frac{-x^2}{2\sigma^2}} \text{ for } x \geq 0 \text{ [3].}$$

If X_1 and X_2 are two independent Gaussian random variables distributed according to $N(m_1, \sigma^2)$ and $N(m_2, \sigma^2)$ (i.e., the variances are equal and the means may be different) then $x = \sqrt{X_1^2 + X_2^2}$ is a Ricean random variable with PDF

$$p_x = \frac{x}{\sigma^2} I_0\left(\frac{sx}{\sigma^2}\right) e^{\frac{-x^2+s^2}{2\sigma^2}} \text{ for } x \geq 0 \text{ [3].}$$

Where 's' is root mean square value of both the means.
Both the Rayleigh distribution and the Rice distribution are frequently used to describe the statistical fluctuations of signals received from a multipath fading channel. Another distribution that is frequently used to characterize the statistics of signals transmitted through multipath fading channels is the Nakagami-m distribution. The PDF for this distribution is given by Nakagami as

$$p_x = \frac{2}{\Gamma(m)}\left(\frac{m}{\Omega}\right)^m x^{2m-1} e^{-mx^2/\Omega} \text{ for } x \geq 0 \text{ [3].}$$

Generalized rician fading unifies both the nakagami-m and Rician fading. For n=2 generalized Rician fading channel becomes Rician fading channel. For n=2m it becomes Nakagami-m fading channel. Generalized Rician fading channel gives better performance than other fading channels.PDF of generalized Rician fading is given by

$$p_x = \frac{x^{n/2}}{\sigma^2 s^{n-2/2}} e^{-\frac{x^2+s^2}{2\sigma^2}} I_{\frac{n}{2}-1}\left(\frac{sx}{\sigma^2}\right) \text{ for } x \geq 0 \text{ [3].}$$

2. Literature Survey

Analysis of average symbol error probability (SEP) for a diversity system over generalized Rician fading channels with correlated branches derived a series expression of the average SEP for a general class of M-ary modulation schemes (including MPSK, MQAM, BFSK, and MSK) with maximal-ratio combining (MRC). The series expression is in canonical form as a weighted sum of elementary closed-form expressions which are the closed-form expressions for the average SEP of a single-branch system in Nakagami fading environments. The analysis was made possible by obtaining the PDF of the instantaneous combiner output SNR as an infinite series of gamma PDFs, and the result could be applied to the special cases of generalized Rayleigh fading, Nakagamiq (Hoyt) fading, generalized gamma fading, Nakagami fading, Rician fading and Rayleigh fading [4]. Statistical properties of generalized Rice multipathfading channels were studied. A closed-form expression for theprobability density function (PDF) of the phase process is derivedfirst. Second order statistics in the form of the level crossingrate (LCR) and the average duration of fades (ADF) arethen investigated for an arbitrary crossing level of the fadingamplitude [5].
Accurate performance evaluation of differentialamplitude and phase-shift keying (DAPSK) with post-detectionequal gain combining (EGC) over generalized fading channelswas of great theoretical interest and practical importance. Using a decision variable-based moment generatingfunction approach, exact error probability results for DAPSK overgeneralized Rician and Nakagami fading channels were derived,taking into account the effects of all the system and fading channelparameters. Several maximum likelihood (ML) based detectorsthat do not require channel state information (CSI) were proposedfor generalized Rician fading channels and an exact bit errorprobability (BEP) union bound for the ML detection was derived.Assuming CSI, a performance upper bound for DAPSK withEGC was also presented. Simulation and numerical results showwhen both detectors had no CSI, the conventional DAPSK detector may perform closely to (though worse than) the ML detector.However, the EGC receiver with CSI performed substantially betterthan the ML detector without CSI and very closely to the coherentAPSK detector [6]. Imperfect channel estimation (ICE) could severelydegrade the bit error rate (BER) of digital modulations withmaximum ratio combining (MRC) diversity reception. The resulting performance analysis problem in its most general setting had not been addressed before. The effect ofICE on the BER of an arbitrary square/rectangular Gray-codedquadratic amplitude modulation (QAM) in generalized Riceanfading channels when MRC reception was employed was analyzed. Ageneral expression for the bit error probability of an arbitrarysquare/rectangular QAM scheme was first derived. This generalformula requires a number of conditional probabilities which wasderived in closed form for independent and non-identically distributed (i.n.d.) Rayleigh-fading channels with MRC and ICE [7].
A framework for analyzing the performance of coded MIMO-OFDM systemsover generalized Rician fading channels is established. Orthogonal space-timeblock codes and receiver maximum ratio combining, and

transmitter beamforming to maximize signal-to-noise ratio at a receiver antenna are assumed. The moment generating function method is applied to obtain the probability of outage,the pairwise error probability, and the bit-error rate of the systems considered.A Prony approximation is shown to be a numerically efficient fading averagingmethod with excellent accuracy. Computationally efficient Prony approximation method turned out to be in excellent agreement with the exact PEP values for all SNR values considered. This fact further validates the space-time-frequency code design rules [8]. Exact expressions for the averageerror performance of M-ary orthogonal signals with non-coherentequal-gain diversity combining over non-identical generalizedRician, Nakagami-m, Nakagami-q and implicitly Rayleigh fadingchannels were derived. The derived expressions were precisely givenin terms of either one-fold integral or rapidly convergent infiniteseries, which could be readily evaluated numerically [9].

Framework for analyzing the performance ofcoded OSTBC-OFDM systems over arbitrary correlated generalized Ricean fading channels was established. The moment generating function of the signal-to-noise ratio at the input to the channeldecoder was derived assuming correlated transmitter and receiverantennas and correlated paths in frequency selective channels.The probability of outage, the pairwise error probability and thebit error rate were then evaluated [10]. A unified analytical framework for evaluating the performance of maximal ratio combining (MRC) and orthogonal space-time block coding (OSTBC) over generalized fading channels was presented. The basic motivation for developing such a framework pertained to analyzing the distribution of a sum of squared random variables (RVs) belonging to different families of fading distributions. Following a novel analytical approach stemming from the definition of a common moment generating function (MGF) model for these families of distributions the probability density function (PDF) and the cumulative distribution function (CDF) of a general sum of squared RVs were expressed by simple infinite Gamma series expansions. Based on these expressions the capacity and error probability of MRC/OSTBC over generalized fading channels were thoroughly studied. The developed theory was used to evaluate the performance of OSTBC for a mixed Nakagami-m/Rice fading model and novel analytical results were presented [11].

The exact error rate analysis for pre-detection equal gain combining over arbitrarily correlated fading branches was elusive. Even specialized correlation models resulted in complex error rate expressions with multiple nested infinite series or integrals. For that reason, asymptotic analysis was a useful tool due to the simplicity of the error rate expressions it produced. However one major shortcoming of this approach was that the asymptotic technique could not predict at what signal-to-noise ratio the asymptotic approximation was accurate. Asymptotically tight single-integral lower and upper bounds on the error probability for a correlated generalized Rician fading model was derived. These lower and upper bounds helped in determining when the asymptotic solutions approached the exact results [12]. Although relatively simple exact error rate expressions were available for selection combining (SC) and equal gain combining (EGC) with independent fading channels, results for correlated channels were highly complex requiring multiple levels of integration when more than two branches were considered. Asymptotic analysis had been used to derive simple error expressions valid in the high signal-to-noise ratio (SNR) region. However it was not clear at what SNR value the asymptotic results were an accurate approximation of the exact solution. Asymptotic results for SC and EGC in correlated generalized. Rician fading channels were derived. Furthermore the asymptotic results for SC were expanded into an exact infinite series. Although this series grew quickly in complexity as more terms were included, truncation to even two or three terms had much greater accuracy than the first (asymptotic) term alone. Finally asymptotically tight lower and upper bounds were derived on the error rate for EGC. These lower and upper bounds helped in determining when the asymptotic solutions approached the exact results [13].

Table 1. Modulation Schemes Comparison

Modulation scheme	Symbol Error Probability
BPSK	$Q\left(\sqrt{2\gamma_b}\right)$

BFSK	$Q\left(\sqrt{\gamma_b}\right)$
QPSK	$2Q\left(\sqrt{\gamma_s}\right)$
4 QAM	$2Q\left(\sqrt{\gamma_s}\right)$

Table 2. Tabular Representation of Literature Survey

Serial no	Title	Author	Publication	Work Done
1	Performance Analysis for Maximal-Ratio Combining in Correlated Generalized Rician Fading	Jay Cheng and Toby Berger	Conference Record of the Thirty-Seventh AsilomarConference on Signals, Systems and ComputersYear: 2003, Vol.2Pages: 1672 - 1675IEEE Conference Publications	Derived expression of the average SEP for M-ary modulation schemes (including MPSK, MQAM, BFSK, and MSK) with MRC over generalized Rician fading channels with correlated branches.
2	On the Statistical Properties of Generalized Rice Multipath Fading Channels	Niji Youssef, Cheng-Xiang Wangt, Matthias Patzoldt, InsafJaafar and Sami Tabbane	Vehicular Technology Conference, VTC 2004 Spring. 2004 IEEE 59th Vol.1. Pages: 162 – 165. IEEE Conference Publications	1.Statistical properties of generalized Rice multipathfading channels were studied. 2.Second order statistics in the form of the level crossingrate (LCR) and the average duration of fades (ADF) were investigated.
3	Diversity Reception Of DAPSK Over Generalized Fading Channels	Yao Ma, Q. T. Zhang And SubbarayanPasupathy	IEEE Transactions On Wireless Communications, Vol. 4, No. 4, July 2005.	1. Exact error probability results for DAPSK overgeneralized Rician and Nakagami fading channels were derived. 2. Assuming CSI, a performance upper bound for DAPSK

				withEGC was also presented.
4	BER Analysis Of Arbitrary QAM For MRC Diversity With Imperfect Channel Estimation In Generalized Ricean Fading Channels	LalehNajafizadeh and ChinthaTellambura	IEEE Transactions On Vehicular Technology, Vol. 55, No. 4, July 2006.	1. A general expression for bit error probability and the effect ofICE on the BER of an arbitrary QAM when MRC reception was employed was analyzed.
5	Performance Analysis Of Coded MIMO-OFDM Systems Over Generalized Ricean Fading Channels	PavelLoskot and Norman C.	Canadian Conference on Beaulieu Electrical and Computer Engineering 2006. CCECE 06. Pages: 1634 – 1639.IEEE Conference Publications	The probability of outage,the pairwise error probability and the bit-error rate was derived for MIMO-OFDM systems.
6	Compact Formulas For The Average Error Performance Of NoncoherentM-Ary Orthogonal Signals Over Generalized Rician, Nakagami-m, And Nakagami-Q Fading Channels With Diversity Reception	Redha M. Radaydeh And Mustafa M. Matalgah	IEEE Transactions On Communications, Vol. 56, No. 1, January 2008.	Exact expressions for the average error performance of M-ary orthogonal signals with non-coherent EGC over non-identical generalized Rician was derived.
7	Approximate Performance Analysis Of Coded OSTBC-OFDM Systems Over Arbitrary Correlated	PavelLoskot And Norman C. Beaulieu	IEEE Transactions on Communications, Vol. 57, No. 8, August 2009.	The probability of outage, the pairwise error probability and the bit error rate of coded OSTBC-OFDM systems over arbitrary

	Generalized Ricean Fading Channels			correlated generalized Ricean fading channels were evaluated.
8	An Exact Performance Analysis of MRC/OSTBC Over Generalized Fading Channels	George A. Ropokis, Athanasios A. Rontogiannis, P. TakisMathiopoulos and Kostas Berberidis	IEEE Transactions On Communications, Vol. 58, No. 9, September 2010.	Exact capacity and error probability analysis forMRC/OSTBC for a mixed over generalized fading channel has been done.
9	Asymptotically Tight Error Rate Bounds For EGC In Correlated Generalized Rician Fading	Josh Schlenker, Julian Cheng And Robert Schober	IEEE ICC Comm. Theory 2013.	Asymptotically tight single-integral lower and upper bounds on the error probability for a correlated generalized Rician fading model was derived.
10	Improving And Bounding Asymptotic Approximations For Diversity Combiners In Correlated Generalized Rician Fading.	Josh Schlenker,Julian Cheng And Robert Schober	IEEE Transactions On Wireless Communications, Vol. 13, No. 2, February 2014.	Asymptotic results for SC and EGC in correlated generalized Rician fading channels were derived.

3. Conclusion

The objective of this paper is to review the progress made so far in Generalized Rician Fading Channel. Generalized Rician fading channel is a recent fading channel. It unifies Rician fading channel and Nakagami-m fading channel. Generalized Rician fading channel gives better performance than other fading channels. Exact error probability analysis for different modulation schemes can be performed using generalized Rician fading channel in future.

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