Performance Analysis of Optimum Combining Under the Impact of Multiple Primary Interferers in Underlay Cognitive Radios

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Abstract-This paper investigates the performance of optimum combining (OC) in underlay cognitive radio (CR) in the presence of multiple primary user's transmitters (PU-Txs) in flat Rayleigh fading channels. The number of PU-Txs L_t and antennas at secondary user receiver (SU-Rx) K_r are related as $L_t \geq K_r$. Closed form expressions are obtained for the ergodic capacity and average bit error rate (BER) of the CR-OC system. Analytical results are validated through simulations. Performance of proposed system is compared with the Maximal ratio combining (MRC) in cognitive radio (CR-MRC).

Keywords: Average bit error rate, Cognitive Radio, Ergodic Capacity, Maximal Ratio Combining, Optimum Combining.

I. INTRODUCTION

Cognitive radio (CR) has been recognized as an efficient solution to deal with the problem of spectrum scarcity (Mitola and Maguire,1999).CR concept allows the secondary user (SU) to access the frequency bands allocated to the primary user (PU) under the condition that the interference generated by SU does not exceed the interference threshold at primary user's receiver (PU-Rx). The capacity under received power constraint at PU-Rx in case of non fading channels was derived in (Gastpar , 2005).The ergodic capacity of SU in several fading environments was investigated in (Ghasemi and Sousa ,2007) and it was concluded that significant capacity gains can be obtained in fading channels as compared to additive white Gaussian noise (AWGN) channels. On the other hand, the performance of SU can be enhanced by employing diversity techniques such as selection combining (SC), maximal ratio combining (MRC), and optimum combining (OC) (Simon and Alouini, 2005). In (Winters,1984) ,it was concluded that the SINR at the output of OC combiner is greater than the output SINR of MRC combiner even when interfering signals are more than the numbers of receive antennas. The MRC in spectrum sharing system was analyzed in (Duan *et al*., 2010) to increase the capacity of SU. In (Huang, 2014), using MRC, the outage performance of SU-Rx is investigated under interference from multiple primary users. However, in all these papers the analysis of MRC in CR is done. But the performance of OC in CR has not been explored well. In this work, we analyze the performance of OC in underlay CR under the impact of interference from multiple PU-Txs in flat Rayleigh fading.

II. SYSTEM MODEL

We consider an underlay CR system which comprises a SU-Tx, a SU-Rx, a PU-Rx and $L_t(l = 1, \dots, L_t)$ PU-Txs .The SU-Rx and PU-Rx consists of $K_r(k = 1, \dots, K_r)$ and $M_r(m = 1, \dots, M_r)$ element antenna arrays, respectively. The SU-Tx and PU-Tx employ single antenna each. The peak interference power constraint, which represents the maximum received interference power at PU-Rx is denoted by Q_p. Let h_s be the $(1 \times K_r)$ channel vector between SU-Tx and SU-Rx, g_s be the $(1 \times M_r)$ channel vector from SU-Tx to PU-Rx and g_{p_l} be the $(1 \times K_r)$ channel vector between l^{th} PU-Tx and SU-Rx. Here the number of PU-Txs and SU-Rx antennas are related as $L_t \geq K_r$ and transmit power of each PU-Tx is assumed to be same. The bandwidth of the system is normalized to 1 and is denoted as B. For the sake of simplicity we neglected the effect of thermal noise. The vector g_{p_l} are independent and identically distributed (i.i.d) complex Gaussian random variables with $E[g_{p_l}] = 0$ and covariance matrix $\epsilon_{cov} = E[g_{p_l} g_{p_l}{}^H]$. These same parameters also hold for h_s. The elements of the vector g_{p_l} have σ^2 variance. We assume that the fading at each antenna element is independent and σ^2=1. The signal transmitted by SU-Tx is BPSK modulated. Assuming that the perfect channel state information (CSI) is

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available at SU-Tx, the maximum permissible transmit power P_s of SU-Tx at each instant is given by

$$P_s = \frac{Q_p}{g_s}$$

(1)

where $g_s = \sum_{m=1}^{M_r} g_{s_m}$ has Chi-Square distribution with $2M_r$ degrees of freedom and its density function is given by

$$f_{g_s}(g_s) = \frac{1}{\Gamma(M_r)} {g_s}^{M_r-1} e^{-g_s}$$

(2)

where Γ is the standard Gamma function and is given as $\Gamma(\alpha) = (\alpha - 1)!$.

Considering the interference from all PU-Txs, the received signal Y_s at the output of antenna elements at SU-Rx can be given as

$$Y_s = \sqrt{P_s}\, h_s x_s + \sum_{l=1}^{L_t} \sqrt{P} g_{p_l} x_{p_l}$$

(3)

where x_s is the signal to be transmitted by SU-Tx and x_{p_l} is the signal of l^{th} PU-Tx. The OC weight vector that maximizes the SIR at the output of antenna array is given by $w = R^{-1}h_s$, where $R = \sum_{l=1}^{L_t} P g_{p_l} g_{p_l}^H$ denotes the interference covariance matrix of all PU-Txs. P denotes the power to be transmitted by l^{th} PU-Tx and the superscript H denotes the complex conjugate transpose.

III. ERGODIC CAPACITY

Using OC weights, the SIR γ_s at the output of antenna elements of SU-Rx can be given by

$$\gamma_s = P_s {h_s}^H R^{-1} h_s = \frac{Q_p}{g_s} {h_s}^H R^{-1} h_s$$

(4)

Let $R = PR_1$,where $R_1 = \sum_{l=1}^{L_t} g_{p_l} g_{p_l}^H$. Therefore, γ_s in (4) becomes

$$\gamma_s = \frac{Q_p}{g_s P} {h_s}^H {R_1}^{-1} h_s = \frac{Q_p}{g_s P} z$$

(5)

where $z = {h_s}^H {R_1}^{-1} h_s$. The density function of random variable z is given by (Shah and Haimovich, 1998)

$$f_z(z) = \frac{\Gamma(L_t+1)}{\Gamma(K_r)\Gamma(L_t+1-K_r)} \frac{z^{K_r-1}}{(1+z)^{L_t+1}}$$

(6)

$z \geq 0, 1 \leq K_r \leq L_t$

The density function in (6) is modified central F Distribution and can be approximated into Chi-Square distribution as (Lee, 2013)

$$f_z(z) = \frac{(L_t+1-K_r)^{K_r}}{\Gamma(K_r)} z^{K_r-1} e^{-(L_t+1-K_r)z}$$

(7)

By substituting $\mu = \frac{z}{g_s}$ in (5) and applying the transformation of two random variables as given in appendix I, we obtain the pdf of ratio of random variables z and g_s as

$$f_\mu(\mu) = \frac{(L_t+1-K_r)^{K_r}}{\Gamma(M_r)\Gamma(K_r)} \mu^{K_r-1} \frac{(K_r+M_r-1)!}{[(L_t+1-K_r)\mu+1]^{K_r+M_r}}$$

(8)

The ergodic capacity of the SU link can be obtained as

$$C_{CR-OC} = \int_0^\infty log_2\left(1+\mu\frac{Q_p}{P}\right) f_\mu(\mu)\, d\mu$$

$$= \frac{(L_t+1-K_r)^{K_r}}{\Gamma(K_r)\Gamma(M_r)} (K_r+M_r-1)! \int_0^\infty \frac{log_2\left(1+\mu\frac{Q_p}{P}\right)\mu^{K_r-1}}{[(L_t+1-K_r)\mu+1]^{K_r+M_r}} d\mu$$

(9)

Applying mathematical modification as in appendix II, we obtain the ergodic capacity of the SU-link as

$$C_{CR-OC} = \frac{(K_r+M_r-1)!}{\Gamma(K_r)\Gamma(M_r)} \sum_{a=0}^{K_r-1} \binom{K_r-1}{a} (-1)^{K_r-1-a} \frac{1}{\log(2)} \frac{1}{(K_r+M_r-a-1)^2} \times {}_2F_1\left(1, K_r+M_r-a-1; K_r+M_r-a, ; \frac{K_r-L_t+\frac{Q_p}{P}-1}{\frac{Q_p}{P}}\right)$$

(10)

where ${}_2F_1$ is the hypergeometric function given as(Andrews,1985)

$${}_2F_1(c,d;e;\zeta) = \sum_{i=0}^\infty \frac{(c)_i (d)_i}{(e)_i} \frac{\zeta^i}{i!}$$

IV. AVERAGE BIT ERROR RATE

For BPSK modulation, the BER evaluated at a given value of γ_s, in terms of Gaussian-Q function is given by (Goldsmith, 2005)

$$P_{e|\gamma_s} = Q\left(\sqrt{2\gamma_s}\right)$$

(11)

The average BER of CR-OC system is obtained by integrating (11) over $f_\mu(\mu)$ .The Q-function and complementary error function are related as $Q\left(\sqrt{2\gamma_s}\right) = \frac{1}{2} erfc\left(\sqrt{\gamma_s}\right)$. Therefore, the average BER of CR-OC system is obtained as

$$P_{e_{CR-OC}} = \frac{1}{2}\int_0^\infty erfc\left(\sqrt{\mu\frac{Q_p}{P}}\right) f_\mu(\mu) d\mu$$

(12)

$$= \frac{1}{2}\frac{(L_t+1-K_r)^{K_r}}{\Gamma(M_r)\Gamma(K_r)} (K_r+M_r-1)! \int_0^\infty erfc\left(\sqrt{\mu\frac{Q_p}{P}}\right) \times \frac{\mu^{K_r-1}}{[(L_t+1-K_r)\mu+1]^{K_r+M_r}} d\mu$$

(13)

$=$

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$$\frac{1}{2\Gamma(M_r)\Gamma(K_r)}\Bigg[\Gamma(M_r)\Gamma(K_r) - \frac{2\sqrt{\frac{Q_p}{P}}\Gamma\left(\frac{-1}{2}+M_r\right)\Gamma\left(\frac{1}{2}+K_r\right)\,{}_pF_q\left(\frac{1}{2},\frac{1}{2}+K_r;\frac{3}{2},\frac{3}{2}-M_r;\frac{Q_p}{P(L_t+1-K_r)}\right)}{\sqrt{\pi(L_t+1-K_r)}} - \frac{(L_t+1-K_r)^{-M_r}\left(\frac{Q_p}{P}\right)^{M_r}\Gamma\left(\frac{1}{2}-M_r\right)\Gamma(K_r+M_r)\,{}_pF_q\left(M_r,M_r+K_r;\frac{1}{2}+M_r,M_r+1;\frac{Q_p}{P(L_t+1-K_r)}\right)}{M_r\sqrt{\pi}}\Bigg] \quad (14)$$

where ${}_pF_q$ is the generalized hypergeometric function (Andrews,1985) .

Hence, (14) is the final expression for the average BER of CR-OC system.

$${}_pF_q(c;d;\zeta) = \sum_{i=0}^{\infty}\frac{(c_1)_i\ldots(c_p)_i}{(d_1)_i\ldots(d_q)_i}\frac{\zeta^i}{i!}$$

V. RESULTS AND DISCUSSION

In this section, analytical as well as simulation results are presented for ergodic capacity and average BER of the proposed system. A good match between simulations and analytical results is observed. The performance of CR-OC system is compared with CR-MRC system. The results presented for MRC are valid for any number of interferers. Here, we assume that the transmit power P of each PU-Tx is 10dB, number of SU-Rx antennas K_r=3 and number of PU-Rx antennas M_r=1. Fig. 1 presents the ergodic capacity of the CR-OC system. As expected, the ergodic capacity becomes poor when PU-Txs are increased. From the figure, we observe that the ergodic capacity of the CR-OC system improves when Q_p is increased i.e the received interference power constraint at PU-Rx is increased which further allows SU-Tx to transmit with increased power. Fig. 2 demonstrates the outcomes of average BER for the CR-OC system. Increase in the value of Q_p leads to an improvement in average BER of the CR-OC system.Fig. 3 and Fig. 4 compares the ergodic capacity and average bit error rate, respectively of both systems system for L_t=3 and L_t=6.

Results indicate that significant capacity gains can be obtained in case of CR-OC as compared to CR-MRC system. It is concluded that when number of PU-Txs and antennas are equal i.e L_t=K_r=3, the performance of CR-OC system outperforms CR-MRC. Whereas for L_t=6 and K_r=3, i.e the number of PU-Tx causing interferers are more than the number of SU-Rx antennas, the performance improvement of CR-OC over CR-MRC becomes small but still OC performs better than MRC. Thus, it is concluded that CR-OC system performs better than CR-MRC system even with the increased number of interferers.

Table 1 shows performance comparison of the CR-OC and CR-MRC system. For L_t=3, 4, 5 and 6 at Q_p=5dB, the ergodic capacity gain of CR-OC is higher than CR-MRC by about 86%, 51%, 38% and 31%, respectively. The ergodic capacity of CR-MRC for L_t=3 is 0.84 bits/s/Hz, whereas the same ergodic capacity is achieved in case of CR-OC for L_t=5.Similarly, the average BER of CR-MRC for L_t=3 is 0.18 and same average BER is obtained for L_t=5 in case of CR-MRC. Thus, it is concluded that CR-OC with two more interferers gives the same performance as CR-MRC. In order to achieve average BER of 10^{-1} at L_t=3 for both the proposed systems, CR-OC requires 5dB less power as compared to CR-MRC. In addition, with L_t=6, for an average BER of $10^{-0.9}$, OC leads to a power saving of nearly 1.5dB over MRC. Thus, it is concluded that the CR-OC is significantly better than CR-MRC even when number of interferers are more than the number of antennas at SU-Rx.

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Parameters	Number of PU-Txs L_t								Notes
	L_t=3		L_t=4		L_t=5		L_t=6		
	CR-OC	CR-MRC	CR-OC	CR-MRC	CR-OC	CR-MRC	CR-OC	CR-MRC	
Ergodic capacity(bits/sec/Hz)	1.56	0.84	1.07	0.71	0.84	0.61	0.71	0.54	Q_p=5dB, P=10dB, M_r=1, K_r=3.
Average bit error rate	0.09	0.18	0.14	0.20	0.18	0.22	0.20	0.24	
$\frac{C_{CR-OC}}{C_{CR-MRC}}$ (%)	86%		51%		38%		31%		

Table1. Comparison of CR-OC and CR-MRC

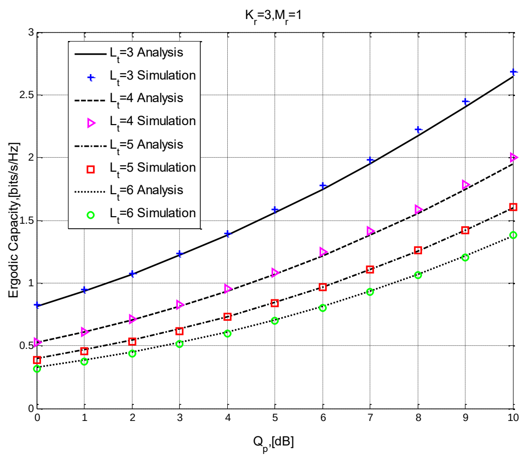

Fig. 1 Ergodic capacity of the CR-OC system versus Q_p for different number of PU-Txs L_t.

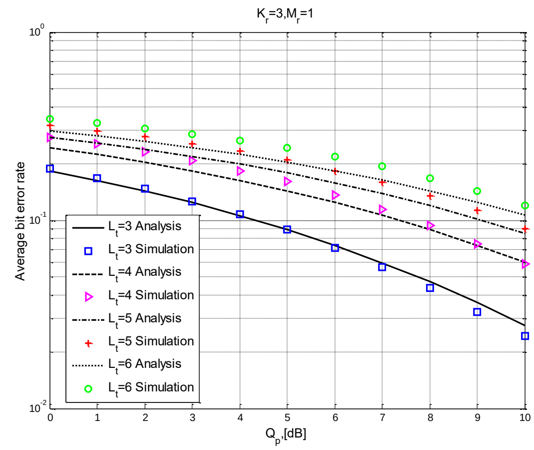

Fig. 2 Average BER of the CR-OC system versus Q_p for different number of PU-Txs L_t.

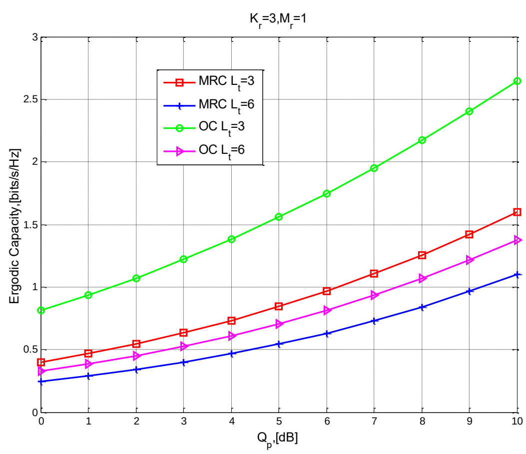

Fig. 3 Performance comparison of ergodic capacity between CR-OC and CR-MRC

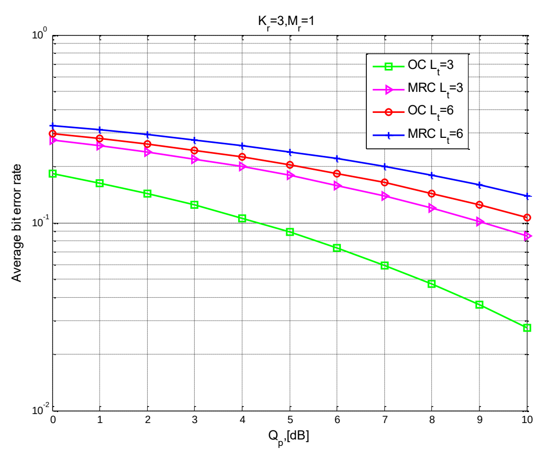

Fig. 4 Performance comparison of average BER between CR-OC and CR-MRC

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VI. CONCLUSION

We analyzed the performance of CR-OC system by taking into account the effect of interference from multiple PU-Txs on the SU-Rx. We analyze the ergodic capacity and average BER of the CR-OC system and observed the effect of different number of PU-Txs on performance of SU-Rx. The performance of CR-OC is then compared to CR-MRC. The results for CR-MRC are presented for an arbitrary number of PU-Txs whereas for CR-OC the number of PU-Txs is greater than number of SU-Rx antennas. We conclude that OC provides better performance as compared to MRC in interference environment. Also we have seen that the CR-OC system leads to much power savings as compared to CR-MRC.

VII. APPENDIX I

Using the substitution $\mu = \frac{z}{g_s}$ and $\beta = g_s$ and using the method to obtain the joint pdf of two independent random variables (Papoulis, 1986), we get

$$f_{\mu,\beta}(\mu,\beta) = |\beta| \times f_z(z) f_{g_s}(g_s)\Big|_{z=\mu\beta, g_s=\beta} \quad \text{(A1)}$$

The marginal pdf of ratio of z and g_s is given as

$$f_\mu(\mu) = \\ = \frac{(L_t+1-K_r)^{K_r}}{\Gamma(M_r)\Gamma(K_r)} \mu^{K_r-1} \int_0^\infty \beta^{K_r+M_r-1} e^{-\beta[(L_t+1-K_r)\mu+1]} d\beta \quad \text{(A2)}$$

When K_r and M_r are integers, the following identity (Gradshteyn and Ryzhik, 1980) can be used to solve the integral in above equation

$$I_1 = \int P_n(v)\, e^{bv}\, dv = \frac{e^{bv}}{b} \sum_{j=0}^{n} \frac{(-1)^j}{b^j} P_n^{(j)}(v) \quad \text{(A3)}$$

Comparing this identity with equation (A2), we get $P_l(\beta) = \beta^{K_r+M_r-1}$, $n = K_r + M_r - 1$ and $b = -[(L_t + 1 - K_r)\mu + 1]$. Using (A3), the integral in (A3) is obtained as

$$\int_0^\infty \beta^{K_r+M_r-1} e^{-\beta[(L_t+1-K_r)\mu+1]} d\beta = \frac{e^{-[(L_t+1-K_r)\mu+1]\beta}}{-[(L_t+1-K_r)\mu+1]} \sum_{j=0}^{K_r+M_r-1} \frac{1}{[(L_t+1-K_r)\mu+1]^j} \times \frac{(K_r+M_r-1)!}{(K_r+M_r-1-j)!} \beta^{K_r+M_r-1-j}\Big|_{\beta=0}^{+\infty}$$

$$= \frac{(K_r+M_r-1)!}{[(L_t+1-K_r)\mu+1]^{K_r+M_r}} \quad \text{(A4)}$$

The final expression for the pdf of μ is obtained by substituting (A4) in (A2) and is given in (8).

VIII. APPENDIX II

To solve (9), substitute $y = (L_t + 1 - K_r)\mu + 1$ and then applying binomial formula, we get

$$C_{CR-OC} = \frac{(K_r+M_r-1)!}{\Gamma(K_r)\Gamma(M_r)!} \sum_{a=0}^{K_r-1} \binom{K_r-1}{a} (-1)^{K_r-1-a} \int_1^\infty log_2\left(1 + \frac{yQ_p}{P(L_t+1-K_r)} - \frac{Q_p}{P(L_t+1-K_r)}\right) y^{a-K_r-M_r} dy \quad \text{(A5)}$$

The above integral can be solved by method of partial integration and is obtained as

$$I_2 = \frac{Q_p}{\log(2) P(K_r+M_r-a-1)} \int_1^\infty \frac{1}{y^{K_r+M_r-a-1}} \frac{1}{(L_t+1-K_r)+y\frac{Q_p}{P}-\frac{Q_p}{P}} dy \quad \text{(A6)}$$

The integral in above equation can be evaluated as

$$I_3 = \frac{1}{\frac{Q_p}{P}(K_r+M_r-a-1)}\ {}_2F_1\left(1, K_r + M_r - a - 1; K_r + M_r - a; \frac{K_r-L_t+\frac{Q_p}{P}-1}{\frac{Q_p}{P}}\right) \quad \text{(A7)}$$

Combining the above three expressions, we obtain the ergodic capacity of CR-OC system and is given by (10).

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