# Effect of External Pressure on the Spherical Shell

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**Abstract**

To grow the prosperity of different spherical shell structures under pressure situations, the present research deals with the examination of elastic - plastic stresses in a spherical shell under the effect of external pressure. The plan of the research has been obtained by using the Seth's transition speculation of elastic- plastic transitions. The transition hypothesis does not accept established suspicions like incompressibility and yield conditions. The radial and circumferential stresses have been computed for the spherical shell for compressible and additionally incompressible materials. It has been watched that the spherical shell made of incompressible material requires high pressure to begin initial yielding in the shell when contrasted with spherical shell made of compressible material. The outcomes inferred are demonstrated graphically.

***Keywords*:** elastic-plastic, pressure, spherical shell, stresses.

## 1. Introduction

Thin shell structures find wide applications in many branches of engineering. Examples include aircraft, spacecraft, nuclear reactors, tanks for liquid and gas storage, and pressure vessels. Engineers have to reduce the weight and cost of the structures, reducing their thickness and applying advanced materials and technologies. This is especially important for aircraft and launch vehicles. Cylindrical and spherical shells are often used as elements in such structures [1]. Engineers have discovered different utilizations of spherical shell structures in aviation, chemical, civil and mechanical ventures, for example, in rapid centrifugal separators, gas turbines for high-control flying machine motors, turning satellite structures, certain rotor frameworks and pivoting magnetic shields. To expand the quality of spherical shells, it is in this way imperative for architects to concentrate the stress investigation in the spherical shells under different conditions. The problems of homogeneous and isotropic spherical shell under pressure have been found in a large portion of the standard elasticity and plasticity books. Evkin *et.al.*[1] suggested asymptotic solution for a thin isotropic spherical shell subject to uniform external pressure and concentrated load. The pressure is the main load and a concentrated lateral load is considered as a perturbation that decreases buckling pressure. The post-buckling solution of the shell under uniform pressure is constructed and discussed buckling behaviors of the spherical shells under uniform pressure. Analyses involved considering the average geometry, average wall thicknesses, and average elastic material properties. Numerical calculations entailed considering the true geometry, average wall thicknesses, and elastic-plastic modeling of true stress–strain curves. Cong *et.al.* [2] discussed the nonlinear axi-symmetric response of shallow spherical FGM shells under mechanical, thermal loads and different boundary conditions based on classical theory of shells. Łukasiewicz [3] derived results for the deformations and stresses under the imposition of concentrated loads in spherical shell. He analyzed his work taking account of stresses normal to the mean surface of the structure and of deformations due to shear stresses. A brief account of methods for designing optimal structures was given by him. Viola *et.al.* [4] studied the static behavior of functionally graded spherical shells and panels subjected to uniform loadings at the extreme surfaces. The material properties are graded in the thickness direction according to a four parameter power law. The structural model involves the a posteriori stress and strain recovery procedure. The obtained governing equations are solved by means of the GDQ numerical technique. An extensive numerical investigation is carried out to characterize the effect of material parameters on the stress, strain and displacement profiles along the thickness direction. These authors have analyzed the problems considering the assumptions (i) incompressibility condition (ii) Creep –strain laws like Norton (iii) Yield condition like that of Tresca (iv) Associated flow rule. The necessity of use of these ad-hoc semi-empirical laws in classical theory of elastic-plastic transition is based on approach that the transition is linear phenomenon which is not possible. Therefore, it suggests that at transition behavior, non-linear terms are significant and cannot be ignored. The concept of generalized strain measures is useful to solve the various problems of elastic -plastic transition by solving the non-linear differential equations at the transition points. This concept of generalized strain measures and transition theory has been applied to find elastic-plastic stresses in various problems; for example Thakur *et.al.*[5-7] analyzed elastic-plastic & creep transition in spherical shell, cylinder and disc with various conditions. All these problems based on the recognition of the transition state as separate state necessitates showing the existence of the constitutive equation for that state. The aim of this paper is to obtain a simple but accurate approximate solution for further analysis of the effect of external pressure on spherical shell. The obtained formulas may also be used for the estimation of stresses at the design state of a real structure under considered load combination. In this paper, we shall derive the results for effective pressure required to start initial yielding in the spherical shell. The stresses under pressure in spherical shell are calculated for compressible as well as for incompressible materials. The results obtained are shown graphically.

## 2. Formulation of the Mathematical Problem

We consider here a thick-walled spherical shell, whose internal and external radii are *a* and *b* respectively, is subjected to uniform external pressure *p*. It is convenient to use spherical polar coordinates $(r,\theta,\phi)$, where $\theta$ the angle *is* made by the radius vector with a fixed axis, and $\phi$ is the angle measured round this axis. By virtue of the spherical symmetry $\sigma_\theta = \sigma_\phi$ everywhere in the shell, due to spherical symmetry of the structure, the components of displacement in spherical co-ordinates $(r,\theta,\phi)$ are given by $u = r(1-\beta), v = 0, w = 0$ where *u*, *v*, *w* (displacement components); $\beta$ is position function, depending on $r = \sqrt{x^2+y^2+z^2}$ only. Generalized components of strain are given by Seth's [8-9]:

$$e_{rr} = \frac{1}{n}\left[1-(r\beta'+\beta)^n\right],\ e_{\theta\theta} = \frac{1}{n}\left[1-\beta^n\right] = e_{\phi\phi}, e_{r\theta} = e_{\theta\phi} = e_{\phi r} = 0, \tag{1}$$

where $\beta' = d\beta/dr$ .

***Stress-Strain Relation:*** The constitutive equation of stress –strain for isotropic material is given as [10]:

$$T_{ij} = \lambda\delta_{ij}I_1 + 2\mu e_{ij}, (i, j = 1, 2, 3) \tag{2}$$

where $\lambda$ and $\mu$ are lame's constants and $I_1 = e_{kk}$ is called first strain invariant.

By using equation (1) in equation (2), the stresses are obtained as:

$$T_{rr} = \frac{\lambda}{n}\left[3 - 2\beta^n - \left(r\beta' + \beta\right)^n\right] + \frac{2\mu}{n}\left[1 - \left(r\beta' + \beta\right)^n\right],$$

$$T_{\theta\theta} = \frac{\lambda}{n}\left[3 - 2\beta^n - \left(r\beta' + \beta^n\right)\right] + \frac{2\mu}{n}\left[1 - \beta^n\right] = T_{\phi\phi},$$

$$T_{r\theta} = T_{\theta\phi} = T_{\phi r} = 0 \qquad (3)$$

***Equation of equilibrium:*** The radial equilibrium of an element of the spherical shell requires:

$$r\frac{dT_{rr}}{dr} - 2\left(T_{\theta\theta} - T_{rr}\right) = 0 \qquad (4)$$

where $T_{rr}$ and $T_{\theta\theta}$ are the radial and circumferential stresses. For sufficiently small values of the pressure, the deformation of the shell is purely elastic. The boundary conditions of problem are

$$T_{rr} = 0 \text{ at } r = a$$

$$T_{rr} = -p \text{ at } r = b, \qquad (5)$$

Using Equations (3) in Equation (4), we get a non- linear differential equation in $\beta$ as:

$$P\left(P+1\right)^{n-1}\beta\frac{dP}{d\beta} + P(P+1)^n + 2(1-c)P - \frac{2c}{n\beta^n}\left[\left\{1 - \beta^n\left(P+1\right)^n\right\} - \left(1 - \beta^n\right)\right] = 0 \qquad (6)$$

where compressibility $c = 2\mu/\lambda + 2\mu$ and $r\beta' = \beta P$ ($P$ is function of $\beta$ and $\beta$ is function of $r$). The transition points of $\beta$ in Equation (6) are $P = 0$, $P \to -1$ and $P \to \pm\infty$. Here by, we are only interested in finding plastic stresses corresponding to $P \to \pm\infty$.

**3. Solution of Problem through Principal Stress:** In order to calculate elastic-plastic stresses, we define the transition function by taking the principal stress $T_{rr}$ (see, Thakur, Verma [11-17]) at the transition point $P \to \pm\infty$.The transition function $R$ is given as:

$$R = 1 - \frac{nT_{rr}}{3\lambda + 2\mu} = \frac{\beta^n}{3-2c}\left[\left(P+1\right)^n + 2\left(1-c\right)\right] \tag{7}$$

Taking the logarithmic differentiating of eq. (7) with respect to $r$ and substituting the value of $dP/d\beta$ from eq. (6) and taking asymptotic value $P \to \pm\infty$, after integration we get:

$$R = A_1 r^{-2c} \tag{8}$$

where $A_1$ is constant of integration.

By using Equations (7) and (8), we have the transition value $T_{rr}$ is

$$T_{rr} = \frac{2\mu(3-2c)}{nc}\left(1 - A_1 r^{-2c}\right) \tag{9}$$

The value of E in the transition range is given by Seth.

$$Y = \frac{E}{n} = \frac{2\mu(1+\sigma)}{n} = \frac{2\mu(3-2c)}{n(2-c)} \tag{10}$$

By using equation (10) and applying boundary condition (i) in equation (9), we have

$$T_{rr} = Y\left(\frac{2-c}{c}\right)\left(1 - \left(\frac{a}{r}\right)^{2c}\right) \tag{11}$$

By using the equation (11) in equation (4)

$$T_{\theta\theta} = T_{rr} + Y(2-c)\left(\frac{a}{r}\right)^{2c} \tag{12}$$

***Initial Yielding:***

It is clear from equation (12) that the value of $\left|T_{\theta\theta} - T_{rr}\right|$ is maximum at $r = b$. Therefore, yielding of the spherical shell take place at the external surface

$$\left|T_{\theta\theta} - T_{rr}\right|_{r=b} = \left|Y(2-c)(a/b)^{2c}\right| \equiv Y_1 \tag{13}$$

External pressure required for initial yielding at the external surface is given by applying boundary condition (ii) in equation (11) as

$$\frac{p}{Y} = \frac{(2-c)}{c}\left\{(t)^{-2c} - 1\right\} \tag{14}$$

where $t = b/a$ denotes thickness ratio of spherical shell.

Radial and circumferential(hoop) stresses for the spherical shell in terms of thickness ratio as

$$T_{rr} = Y\left(\frac{2-c}{c}\right)\left(1 - \left(\frac{rt}{b}\right)^{-2c}\right)$$

$$T_{\theta\theta} = T_{rr} + Y(2-c)\left(\frac{rt}{b}\right)^{-2c} \tag{15}$$

***Fully-Plastic state:***

For fully plastic state, we make c→0 in equations (14, 15) and we get the following equations

$$P_f = \frac{p}{Y} = 4\log(1/t) \tag{16}$$

$$T_{rr} = 4Y\log(b/rt) \tag{17}$$

$$T_{\theta\theta} = T_{rr} + 2Y \tag{18}$$

Now we convert the above equations for initial yielding and fully plastic state in non-dimensional form by introducing the following non-dimensional components as

$$\sigma_r = \frac{T_{rr}}{Y}, \sigma_\theta = \frac{T_{\theta\theta}}{Y}, R = \frac{r}{b}, P_i = \frac{p}{Y} \tag{19}$$

Therefore, the expressions for transitional stresses and pressure are given for initial yielding

$$P_i = \frac{(2-c)}{c}\left\{(t)^{-2c} - 1\right\} \tag{20}$$

$$\sigma_r = \left(\frac{2-c}{c}\right)\left(1-\left(Rt\right)^{-2c}\right)$$

$$\sigma_\theta = \sigma_r + (2-c)\left(Rt\right)^{-2c} \tag{21}$$

In case of fully Plastic state, the expressions in non-dimensional form become

$$P_f = 4\log(1/t) \tag{22}$$

$$\sigma_r = 4\log(1/Rt) \tag{23}$$

$$\sigma_\theta = \sigma_r + 2 \tag{24}$$

**9.2.4 RESULTS AND DISCUSSIONS**

From the above investigation of derived results, the external pressure is figured for the different value of compressibility. Pressure values are calculated for thinner and thicker spherical shells. It is clear from equations (22) and (23) that pressure obtained is negative. This shows that external pressure is compressible in nature. For sake of convenience, we ignore the negative sign while representing the pressure in Fig.1 and Fig.2. Curves are plotted between pressure along the compressibility factor *c* (see Fig.1) for the spherical shells with the different thickness ratio. It has been watched that the spherical shell made of low compressibility material requires high pressure to begin initial yielding in the shell when contrasted with spherical shell made of compressible material with *c* = 0.50, 0.75. Also impact of thickness ratio is seen on the spherical shell *i.e.* thicker shells require high pressure to yield as compared to the thinner shells. In case of fully plastic state, the role of compressibility vanishes and thicker spherical shells attain fully plastic state as compared to the thinner spherical shell. In Fig.2, curves are drawn for pressure required to attain fully plastic state against the compressibility factor. Pressure calculated is independent of compressibility of the material at fully plastic state. Radial and hoop stresses are shown along the radii ratio for spherical shell made up of compressible and incompressible materials(see Fig.3 & Fig.4). It is seen that the value of hoop stresses are maximum at the external surface of spherical as compared to the radial stresses. With the effect of thickness ratio, the value of radial and hoop stresses is increased at the external surface of the shell. In fully plastic state , the stress distribution is reversed and it is found to be maximum at the internal surface of the spherical shell(see Fig.5).

### 9.4.5 Conclusion:

The problems' solution in paper concludes that spherical shell made up of incompressible material is the secure design in contrast to the spherical shell made up of highly compressible material under pressure. It happens due to high pressure requisite to start initial yielding in the spherical shell made up of incompressible material which leads to the safety and durability of the spherical shell.

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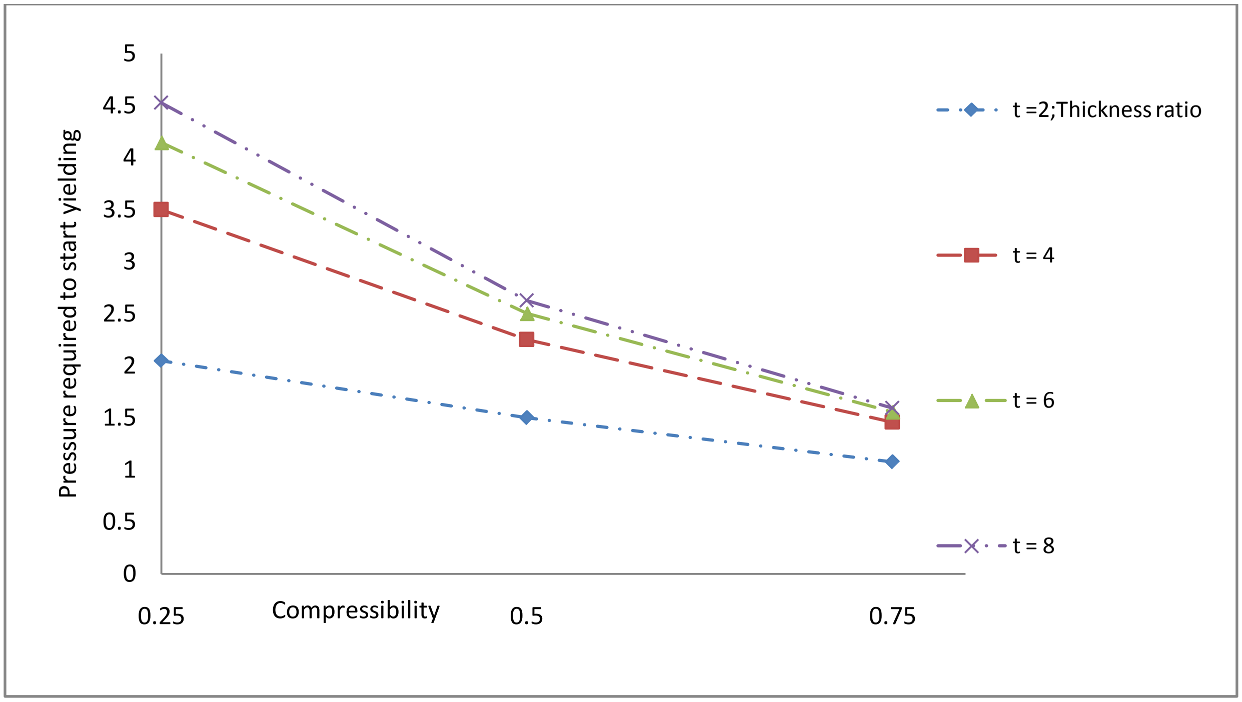


Fig.1: Pressure required to start yielding in spherical shell in dependence of the compressibility

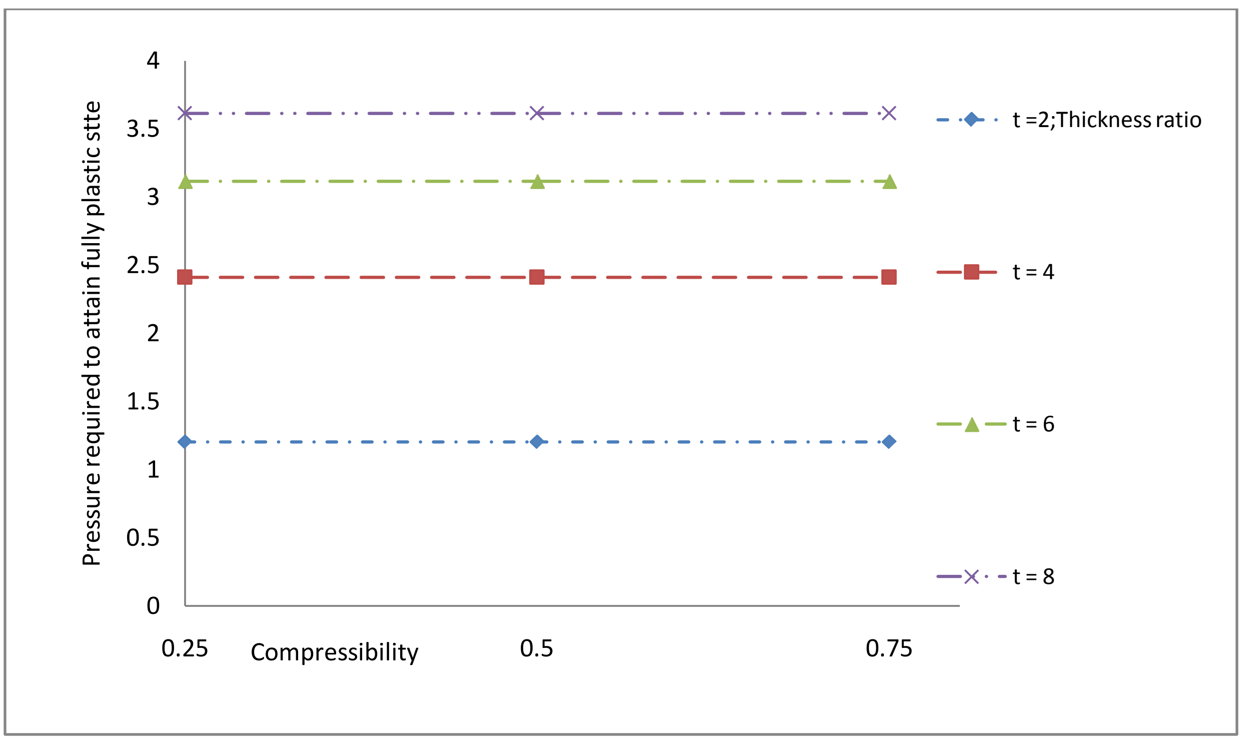

Fig.2: Pressure required to attain fully plasticity in spherical shell in dependence of the compressibility

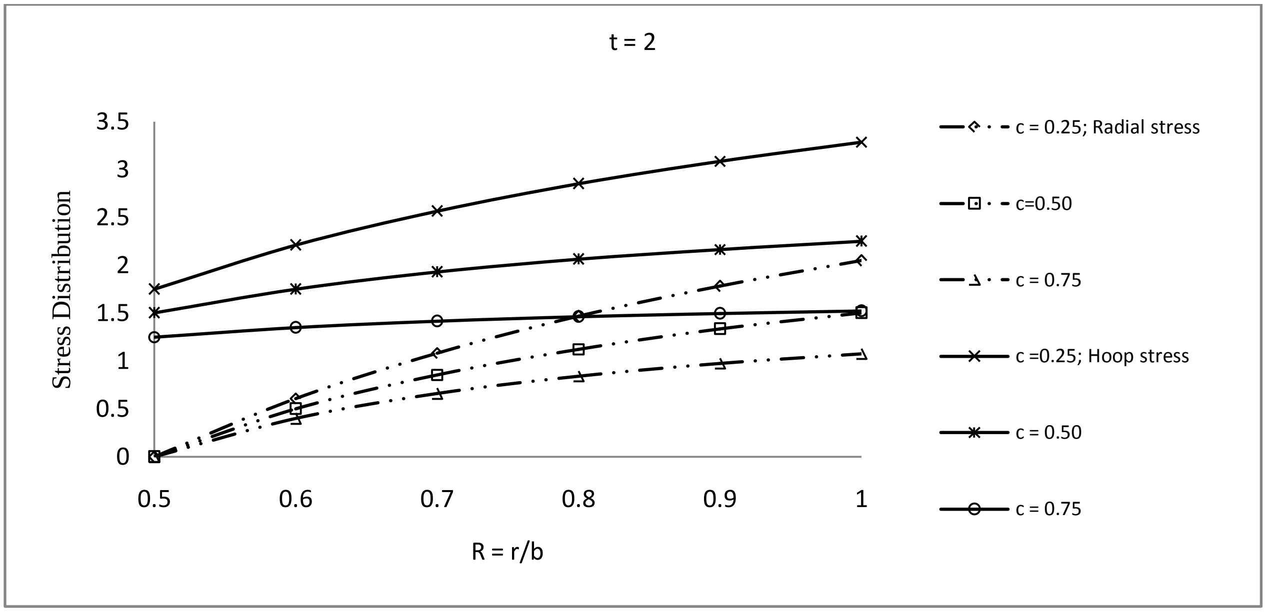


Fig.3: Stress distribution in spherical shell in dependence of compressibility for $t = 2$

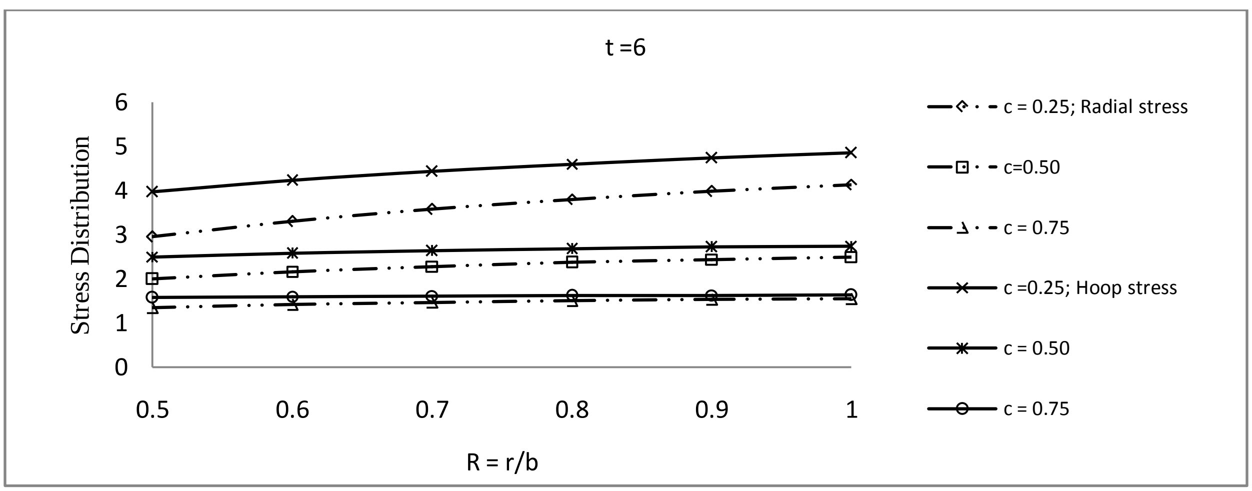


Fig.4: Stress distribution in spherical shell in dependence of compressibility for $t = 6$

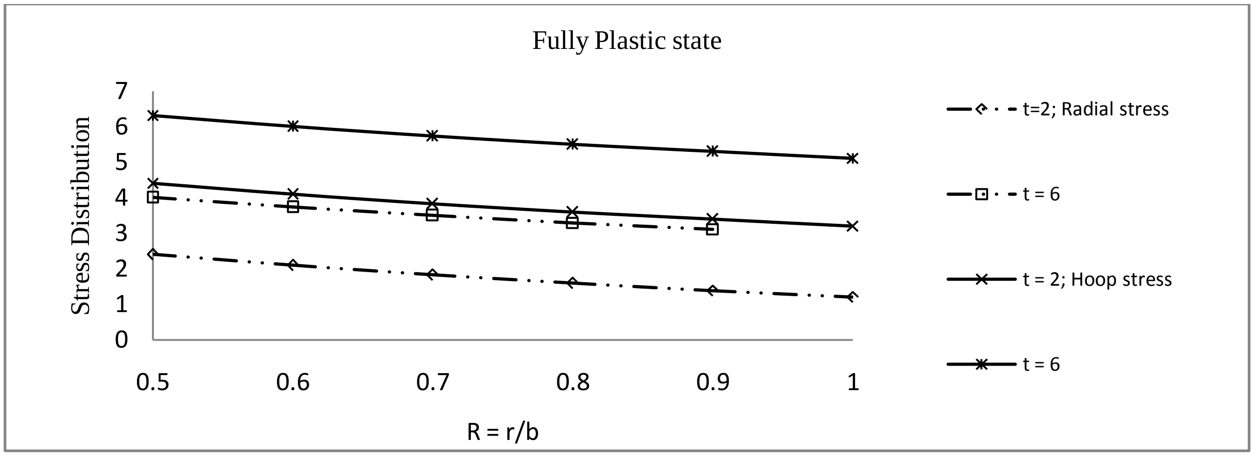


Fig.5: Stress distribution in spherical shell in dependence of compressibility for fully plastic state

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*ISSN: 2229-6913(Print), ISSN: 2320-0332(Online), UGC Approved Journal (S.No.63019)*
*Web Presence: http://ijoes.vidyapublications.com*