Generation of Planar Kinematic Chains

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Abstract
Kinematic chains constitute the basis for developing all kinds of mechanisms. M.Gruebler in his classical work Getriebelehre (kinematics) found a relationship between the number of links (n),the number of joints (j) and the degree of freedom (f) of a kinematic chain.In this paper, a simple, efficient and reliable computational method to develop all possible distinct planer, pin-jointed, 1-dof, KC up to 10-link kinematic chains from a given assortment of links for single degree of freedom has been proposed.The entire skeleton and KC are distinct as the permutation process of E-; Z- and D-Chains is rigorous. The method is based on theoretic approach. All the possible skeletons of kinematic chains (KC) having 6-links and 8-links 1-dof have been developed. This work will be extremely useful and give the freedom for new researchers / designers to select the best mechanism kinematic chain and the mechanism to be used to perform the end task according to needs at the conceptual stage of design.

Keywords:
Kinematic Chain; Skeleton; E-; Z-; D- and V-Chains

1. Introduction

A number of researchers have discussed structural synthesis in the earlier days. Crossley [1] proposed a collection of 10-link plane chains. During the compiling of this collection, his greatest problem was to distinguish whether two arrangements, which might appear unlike, were actually the same or different. This led to a definition of isomorphism between linkages.. Mruthyunjaya [2] made an effort to develop a fully computerized approach for structural synthesis of kinematic chains. Nageswara and Rao [3] investigated selection of best frame in function generators. Sethi and Agrawal [4] proposed a classification scheme on the basis of structural properties. Madan and Jain [5] considered the kinematic chains-isomorphism, inversions and degree of similarity using the concept of connectivity. Rao [6] threw the light on the enumeration of distinct planar kinematic chains. They developed a very simple method based on independent loop(s) assorted and their adjacency is reported. Misti [7] presented the position analysis in polynomial form of planar mechanisms with Assur groups of class 3 including revolute and prismatic joints. Uicker and Raicu [8] presented a method for the identification and recognition of equivalence of kinematic chains. Later on, this method failed. Mruthyunjaya and Balasubramanian [9] proved that the method proposed by Uicker and Raicu [8] is not reliable. They proved that the test based on comparison of the characteristic coefficients of the adjacency matrices of the corresponding graphs for detection of isomorphism in kinematic chains failed. Shende and Rao [10] work, which deals with the problem of detection of isomorphism which is frequently encountered in structural synthesis of kinematic chains. Chu Jin-Kui and Cao Wei-Qing [11] proposed a method for identification of isomorphism among kinematic chains and inversions using Link's adjacent-chain-table. Yadav, et.al. [12] Proposed a computer aided detection method of isomorphism among kinematic chains and mechanisms using the concept of modified distance. Yadav, et.al.[13] presented a paper mechanism of a kinematic chain and the degree of structural similarity based on the concept of link path code'. Yadav, et.al.[14] presented a paper 'computer aided detection of isomorphism among binary chains using the link-link multiplicity distance concept. Rao [15] suggested the application of fuzzy logic for the study of isomorphism, inversions, symmetry, parallelism and mobility in kinematic chains with some necessary and sufficient conditions. Kong, et.al. [16] Proposed a new method based on artificial neural network (ANN) to identify the isomorphism of the mechanism kinematic chain. Rao and Deshmukh [17] proposed method does not require any separate test for isomorphism in the generation of kinematic chains. Chang, et.al. [18] proposed method is based on the eigen vectors and eigen values to identify isomorphism of mechanism kinematic chain.. He and Jhang [19] proposed a new method for detection of graph isomorphism based on the quadratic form. Tang and Liu [20] established a method 'the degree code' as a new mechanism identifier. Later on this method also failed. Zhao, et.al [21] put forward and more complete theory of degrees of freedom (DOF) for mechanisms, especially for the complex spatial mechanisms, which may not be solved correctly with traditional theories. Jensen [22] gave a systematic mechanism design for engineers and inventore. Hasan [23-24] proposed a new method in which kinematic chains are represented in the form of the Joint-Joint [JJ] matrix. Two structural invariants, sum of absolute characteristic polynomial coefficients and maximum absolute value of the characteristic polynomial coefficient are derived from the characteristic polynomials of the [JJ] matrix of the kinematic chains. Dargar et al. [25-26] proposed Link adjacency value method to identify the isomorphism by calculating the first and second link adjacency values. Rizvi et al. [27] presented a new method for distinct inversions and isomorphism based on a link identity matrix and link signature. Alam et al.[28] presented weighted squared path technique to determine the structural similarity and dissimilarity in the kinematic chains.

2. Definitions

E-Chain: A chain, in which two polygonal links are directly connected with one joint.

Z-Chain: A chain, in which two polygonal links are connected with the help of one intermediate binary link.

D-Chain: A chain, in which two polygonal links are connected with the help of two intermediate binary links.

V-Chain: A chain, in which two polygonal links are connected with the help of three intermediate binary links.

Representation of Polygonal Links: For abstract representation, each ternary, quarternary, ----, n-nary link is represented by a circle surrounding '3','4', -, -, -, 'n' respectively, radiating the lines equal to the surrounding number within the circle.

Skeleton:

It is the representation of kinematic structures in abstract form. The connections between polygonal links are made with the help of strings of binary links and designated as E-, D-, Z- and V-Chains.

Connection, C= total number of E-, Z-, D- and V-Chains existing in a KC.

½ [Sum of radiated rays from the circles]

i_{max}

$$= \frac{1}{2} \sum i.ni \quad (1)$$

Where:nı i=3 r of i-gonal links. For ternary links, ni=3.For quaternary links, ni=4, etc.,

i_{max}= n / 2

3. Calculations of E-, Z-, D- and V-Chains

To obtain the distinct KC from a given skeleton, the number of **E**-, Z-, D- and V-Chains should satisfy the following equations. (i) The sum of number E-, Z-, D- and V-Chains should be equal to the number of connections C present in the given skeleton is given by eq. (2). (ii) We know that the number of binary links in E-, Z-, D- and V- chains are 0,1,2 and 3 respectively. so the total number of binary links should be equal to the binary links attached to the E-, Z-, D- and V- chains, given by eq. (3). (iii) the number of joints in E-, Z-, D- and V-Chains are 1, 2, 3 and 4 respectively andis given by eq. (4).

$$E+Z+D+V = C \quad (2)$$

$$n_2 = 0 \times E + 1 \times Z + 2 \times D + 3 \times V = Z+2D+3V \quad (3)$$

$$j = = 1 \times E + 2 \times Z + 3 \times D + 4 \times V \quad (4)$$

As we know that the KC up to 10-links have E-, Z-, and D- Chains only.i.e.V=0 [3]. The maximum number of E-chains existing in a KC may be equal to the number of polygonal links plus one and minimum value may be equal to the difference between number of connections C and number of binary links. These are given by eq. (5) and (6). The number of E-chains in skeletons of KC may vary from E_{max} to E_{min} only. For each value of E-chains, we find the number of Z-and D-chains for V=0. Therefore, for each considered value of E-chain and V=0, the equ. (2), (3) and (4) may be written as eq. (7), (8) ,(9) and after solving these equations with the help of matrix representation ,we get eq.(10) and (11). On solving the equations (10) and (11), we get eq. (12) and (13). Eq. (12 and (13) are used directly to calculate the number of D- and Z – Chains for each value of E from E_{max} to E_{min} and V=0. The next step in final development of skeletons for the possible combinations of E-, Z- and D-chains is to find out what possibilities exist to replace the circle connections of skeletons with E-, Z-, and D- chains. The procedure of permutation of E-, Z-, and D- chains is explained in detail in illustrative example.

$$E_{max} = n_3 + n_4 + \text{-------------} + n_{max+1} \quad \text{------------ (5)}$$

$$E_{min} = C - n_2 \quad \text{--------------------------- ---(6)}$$

$$Z+D = (C-E) \quad \text{------------------------------ (7)}$$

$$Z+2D = n_2 \quad \text{------------------------------ (8)}$$

$$2Z+3D = (j-E) \quad \text{----------------------------- (9)}$$

$$6Z+9D = [(C-E) + n_2 + 2(j-E) \quad \text{-------------------(10)}$$

$$9Z+14D = [(C-E) + 2n_2 + 3(j-E) \quad \text{--------------- (11)}$$

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$D = n_2 - C+E$ ---------------------------- (12)

$Z= C-E-D$ ------------------------------------ (13)

4. Methodology

(i) Find the maximum degree of link using: i_{max} =n/2 ,Where n= total no. of links. (ii)Determine the connections (C) using equation (1).(iii)Determine the maximum and minimum number of E-Chains using eq. (5) (6).(iv)Determine the number of D-and Z-chains using equation (12) and (13) for E_{max} to E_{min} and obtain all possible solution set of E-, Z- and D-Chains. If all the values of E-, Z- and D-Chains are positive then solution will exist. But if the value of any one (of E-, Z-, and D-) chain is negative, then solution will not exist.**(v)** Develop the skeleton of KC by permutation of E-, Z- and D-Chains.

5. Illustrative Example

The example concerns class-III (a) 1-F, KC in which n =8, n_2=4, n_3=4.we have to develop distinct KC from this combination.

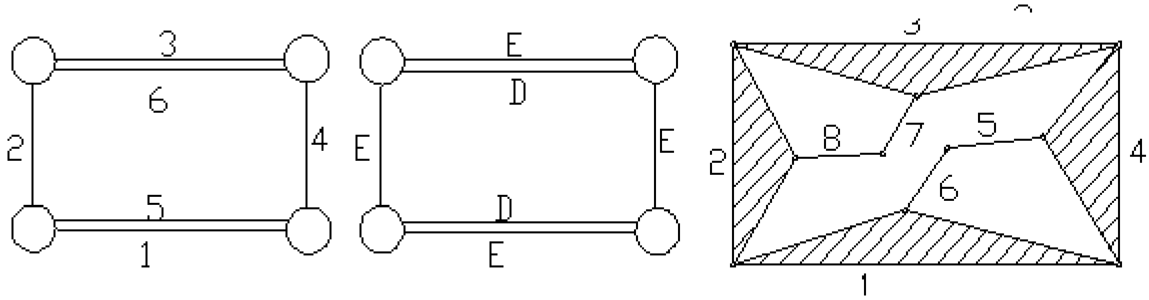
Maximum degree of link is: i_{max} = n/2 =8/2=4. The number of connections (C) using Eq. (1), C= ½[4 x 3 + 4 x 0] =6. E_{max} = 4 + 0 + 1= 5 and E_{min} = 6-4=2. It means the number of E-Chains varies from 2 to 5.there may be four cases. (a) E =5, (b) E = 4, (c) E = 3, (d) E =2. Now, the number of D-and Z-Chains for each case of E_{max} to E_{min} are determined using Eq. (12) and Eq. (13) respectively.

For case (a) when E=5 : D = n_2-C+E = 4-6+5 =3 and Z= C-E-D = 6-5-3= -2. Here the values of Z- are negative. Therefore, solution does not exist. For Case (b) when E =4: D = n_2-C+E = 4-6+4 = 2 and Z= C-E-D = 6 – 4 – 2 = 0. Since the values of E-, D- and Z- are either zero or greater than zero. Therefore, solution exists. Hence first possible solution set is E =4, Z= 0 and D=2.For Case (c) when E=3: Using the same procedure, second possible solution set is E =3, Z=2 and D=1.For Case (d) when E=2: The third possible solution set is E =2, Z=4 and D=0.

The Permutation of E-, Z-and D-Chains in a Skeleton of Kinematic Chains.

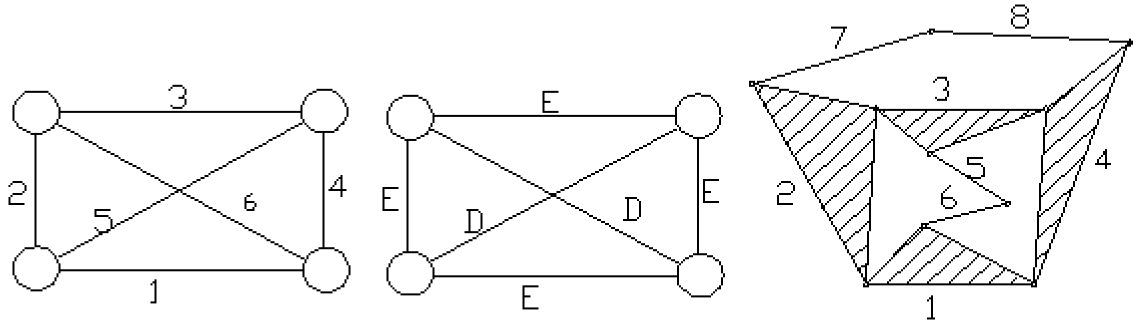
Considering first possible solution set for which n_2 =4, n_3 =4, C=6, E =4, Z=0 and D=2.Now, we have to develop all possible skeleton containing 4E-Chains and 2D-Chains.The 4E-Chains are permuted first. The 4E-Chains can be permuted in four connections numbered 1, 2, 3 and 4 by only one way in the skeleton as shown in Fig.1 (a). The 2D-Chains are now can be attached with multiple connections numbered as 5 and 6. Hence the connections of the skeleton of Fig.1 (a) are replaced by 4 E-Chains and 2D-Chains as shown in Fig.1 (b). Now the KC is developed from this skeleton as shown in Fig.1(c).

Similarly, another arrangement of skeleton for the first possible solution set is shown in Fig.2 (a). The numbers of E-, Z- and D-Chains are same i.e. 4, 0, and 2 respectively. For this arrangement, only one combination 1234 can be used to attach 4E-Chains. The 2D-Chains are then attached to diagonal connections numbered as 5 and 6. The final skeleton and its distinct KC are shown in Fig.2(b) and Fig.2(c) respectively. Hence, Following the same procedure the development of skeleton and KC for second set and third set of solutions can be carried out.



(a) Connections (b) Skeleton (c) 8-link, 10-joint, 1-dof KC

Fig.1: Representation of 'C', Skeleton and KC after permutation of E-, Z- and D-Chains
(n_2 = 4, n_3 =4, C=6, E=4, Z =4, D=2)



(a)Connections (b)Skeleton (c) Kinematic chain

Fig.2: Representation of 'C', Skeleton and KC after permutation of E-, Z- and D-Chains
(n_2 = 4, n_3 =4, C=6, E=4, Z =4, D=2)

Using the same procedure, the skeleton and distinct KC may be developed for other possible solution sets, which are summarized in Table-1.

6. Precautions

The following combination of E- and Z-chains cannot be used because the result is a rigid structure. Two E- chains cannot be attached between two polygonal links.(ii)the combination of E- and Z-chains cannot be attached between two polygonal links. (iii) Three Z-chains cannot be attached simultaneously between two polygonal links. In all the cases mentioned above the dof ≤ 0.

7. Results

The detailed results of the developed distinct skeletons and KC derived from 1-dof, 6-links and 8-links are available with the author. Using the proposed method the number of distinct KC generated from 1-dof, 6-links and 8-links are 2 and 16 respectively. These results agree as found in the literature [22].

8. Conclusions

The proposed method incorporates all features of the KC.The permutation process takes care the elimination of rigid structures and non-isomorphism among the generated skeletons and KC. Generation of KC by Mechanism for Engineers [7]) is an exhaustive process while in the proposed method simple one. The future scope of this work may be to extend the process for KC having 10 and more than 10-links. The process may be correlated with the joint performance like compactness, wear, and stiffness of the KC.

Table 1: Possible sets of E-, Z-, D-Chains, and distinct KC for 6-link and 8-links 1-dof.

Class	Type of Link				Conn. C	Possible Solution set	Types of Chains			No. of solutions	Total Solutions	Distinct KC
	n_2	n_3	n4	n5			E	Z	D			
II(a)	4	2	-	-	3	I	1	0	2	1	2	2
						II	0	2	1	1		
III (a)	4	4	0	-	6	I	4	0	2	2	9	16
						II	3	2	1	4		
						III	2	4	0	3		
III (b)	5	2	1	-	5	I	2	1	2	2	5	
						II	1	3	1	2		
						III	0	5	0	1		
III(c)	6	0	2	-	4	I	1	0	3	1	2	
						II	0	2	2	1		
						II	1	0	4	1		

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